

# CHAPTER 20

## SIGHT REDUCTION

### BASIC PRINCIPLES

#### 2000. Introduction

Reducing a celestial sight to obtain a line of position consists of six steps:

1. Correcting sextant altitude ( $h_s$ ) to obtain observed altitude ( $h_o$ ).
2. Determining the body's GHA and declination.
3. Selecting an assumed position and finding that position's local hour angle.
4. Computing altitude and azimuth for the assumed position.
5. Comparing computed and observed altitudes.
6. Plotting the line of position.

This chapter concentrates on using the *Nautical Almanac* and *Pub. No. 229, Sight Reduction Tables for Marine Navigation*.

The introduction to each volume of the *Sight Reduction Tables* contains information: (1) discussing use of the publication in a variety of special celestial navigation techniques; (2) discussing interpolation, explaining the double second difference interpolation required in some sight reductions, and providing tables to facilitate the interpolation process; and (3) discussing the publication's use in solving problems of great circle sailings. Prior to using the *Sight Reduction Tables*, carefully read this introductory material.

Celestial navigation involves determining a circular line of position based on an observer's distance from a celestial body's geographic position (GP). Should the observer determine both a body's GP and his distance from the GP, he would have enough information to plot a line of position; he would be somewhere on a circle whose center was the GP and whose radius equaled his distance from that GP. That circle, from all points on which a body's measured altitude would be equal, is a **circle of equal altitude**. There is a direct proportionality between a body's altitude as measured by an observer and the distance of its GP from that observer; the lower the altitude, the farther away the GP. Therefore, when an observer measures a body's altitude he obtains an indirect measure of the distance between himself and the body's GP. Sight reduction is the process of converting that indirect measurement into a line of position.

Sight reduction reduces the problem scale to manageable size. Depending on a body's altitude, its GP could be thousands of miles from the observer's position. The size of

a chart required to plot this large distance would be impractical. To eliminate this problem, the navigator does not plot this line of position directly. Indeed, he does not plot the GP at all. Rather, he chooses an **assumed position (AP)** near, but usually not coincident with, his DR position. The navigator chooses the AP's latitude and longitude to correspond to the entering arguments of LHA and latitude used in the *Sight Reduction Tables*. From the *Sight Reduction Tables*, the navigator computes what the body's altitude *would have been* had it been measured from the AP. This yields the **computed altitude ( $h_c$ )**. He then compares this computed value with the **observed altitude ( $h_o$ )** obtained at his actual position. The difference between the computed and observed altitudes is directly proportional to the distance between the circles of equal altitude for the assumed position and the actual position. The *Sight Reduction Tables* also give the *direction* from the GP to the AP. Having selected the assumed position, calculated the distance between the circles of equal altitude for that AP and his actual position, and determined the direction from the assumed position to the body's GP, the navigator has enough information to plot a line of position (LOP).

To plot an LOP, plot the assumed position on either a chart or a plotting sheet. From the *Sight Reduction Tables*, determine: 1) the altitude of the body for a sight taken at the AP and 2) the direction from the AP to the GP. Then, determine the difference between the body's calculated altitude at this AP and the body's measured altitude. This difference represents the difference in radii between the equal altitude circle passing through the AP and the equal altitude circle passing through the actual position. Plot this difference from the AP either *towards* or *away from* the GP along the axis between the AP and the GP. Finally, draw the circle of equal altitude representing the circle with the body's GP at the center and with a radius equal to the distance between the GP and the navigator's actual position.

One final consideration simplifies the plotting of the equal altitude circle. Recall that the GP is usually thousands of miles away from the navigator's position. The equal altitude circle's radius, therefore, can be extremely large. Since this radius is so large, the navigator can approximate the section close to his position with a straight line drawn perpendicular to the line connecting the AP and the GP. This straight line approximation is good only for sights of relatively low altitudes. The higher the altitude, the shorter the distance between the GP and the actual position, and the

smaller the circle of equal altitude. The shorter this distance, the greater the inaccuracy introduced by this approximation.

### 2001. Selection Of The Assumed Position (AP)

Use the following arguments when entering the *Sight Reduction Tables* to compute altitude ( $h_c$ ) and azimuth:

1. Latitude (L).
2. Declination (d or Dec.).
3. Local hour angle (LHA).

Latitude and LHA are functions of the assumed position. Select an AP longitude resulting in a whole degree of LHA and an AP latitude equal to that whole degree of latitude closest to the DR position. Selecting the AP in this manner eliminates interpolation for LHA and latitude in the *Sight Reduction Tables*.

Reducing the sight using a computer or calculator simplifies this AP selection process. Simply choose any convenient position such as the vessel's DR position as the assumed position. Enter the information required by the specific celestial program in use. Using a calculator reduces the math and interpolation errors inherent in using the *Sight Reduction* tables. Enter the required calculator data carefully.

### 2002. Comparison Of Computed And Observed Altitudes

The difference between the computed altitude ( $h_c$ ) and the observed altitude ( $h_o$ ) is the **altitude intercept** (a).

The altitude intercept is the difference in the length of

the radii of the circles of equal altitude passing through the AP and the observers actual position. The position having the greater altitude is on the circle of smaller radius and is closer to the observed body's GP. In Figure 2003, the AP is shown on the inner circle. Therefore,  $h_c$  is greater than  $h_o$ .

Express the altitude intercept in nautical miles and label it T or A to indicate whether the line of position is toward or away from the GP, as measured from the AP.

A useful aid in remembering the relation between  $h_o$ ,  $h_c$ , and the altitude intercept is: H<sub>o</sub> M<sub>o</sub> T<sub>o</sub> for H<sub>o</sub> More Toward. Another is C-G-A: Computed Greater Away, remembered as Coast Guard Academy. In other words, if  $h_o$  is greater than  $h_c$ , the line of position intersects a point measured from the AP towards the GP a distance equal to the altitude intercept. Draw the LOP through this intersection point perpendicular to the axis between the AP and GP.

### 2003. Plotting The Line Of Position

Plot the line of position as shown in Figure 2003. Plot the AP first; then plot the azimuth line from the AP toward or away from the GP. Then, measure the altitude intercept along this line. At the point on the azimuth line equal to the intercept distance, draw a line perpendicular to the azimuth line. This perpendicular represents that section of the circle of equal altitude passing through the navigator's actual position. This is the line of position.

A navigator often takes sights of more than one celestial body when determining a celestial fix. After plotting the lines of position from these several sights, advance the resulting LOP's along the track to the time of the last sight and label the resulting fix with the time of this last sight.

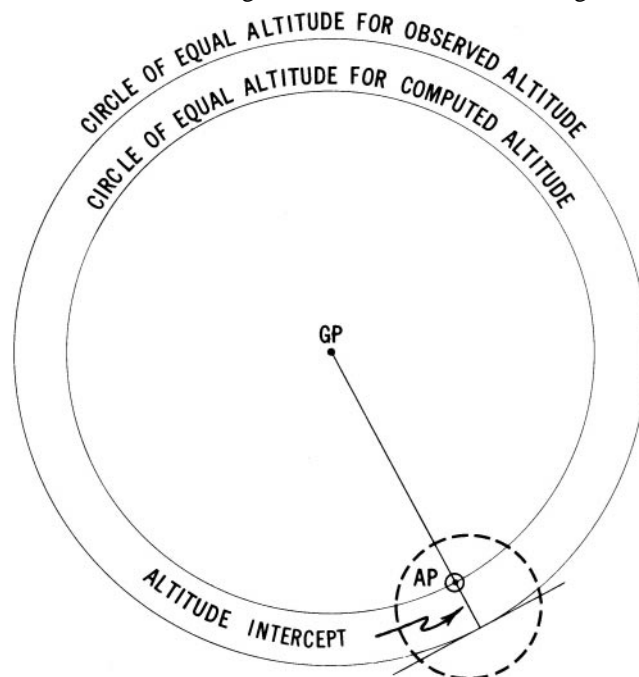


Figure 2003. The basis for the line of position from a celestial observation.

### 2004. Recommended Sight Reduction Procedure

Just as it is important to understand the theory of sight reduction, it is also important to develop a working procedure to reduce celestial sights accurately. Sight reduction involves several consecutive steps, the accuracy of each completely dependent on the accuracy of the steps that went before. Sight reduction tables have, for the most part, reduced the mathematics involved to simple addition and subtraction. However, careless errors will render even the most skillfully measured sights inaccurate. The navigator must work methodically to reduce these careless errors.

Naval navigators will most likely use OPNAV 3530, U.S. Navy Navigation Workbook, which contains pre-formatted pages with “strip forms” to guide the navigator through sight reduction. A variety of commercially-produced forms are also available. Pick a form and learn its method *thoroughly*. With familiarity will come increasing understanding.

Figure 2004 represents a functional and complete worksheet designed to ensure a methodical approach to any sight reduction problem. The recommended procedure discussed below is not the only one available; however, the navigator who uses it can be assured that he has considered *every* correction required to obtain an accurate fix.

**SECTION ONE** consists of two parts: (1) Correcting sextant altitude to obtain apparent altitude; and (2) Correcting the apparent altitude to obtain the observed altitude.

**Body:** Enter the name of the body whose altitude you have measured. If using the sun or the moon, indicate which limb was measured.

**Index Correction:** This is determined by the characteristics of the individual sextant used. Chapter 16 discusses determining its magnitude and algebraic sign.

**Dip:** The dip correction is a function of the height of eye of the observer. It is always negative; its magnitude is determined from the Dip Table on the inside front cover of the *Nautical Almanac*.

**Sum:** Enter the algebraic sum of the dip correction and the index correction.

**Sextant Altitude:** Enter the altitude of the body measured by the sextant.

**Apparent Altitude:** Apply the sum correction determined above to the measured altitude and enter the result as the apparent altitude.

**Altitude Correction:** Every observation requires an altitude correction. This correction is a function of the apparent altitude of the body. The *Almanac* contains tables for determining these corrections. For the sun, planets, and stars, these tables are located on the inside front cover and facing page. For the moon, these tables are located on the back inside cover and preceding page.

**Mars or Venus Additional Correction:** As the name implies, this correction is applied to sights of Mars and Venus. The correction is a function of the planet measured, the time of year, and the apparent altitude. The inside front cover of the *Almanac*

lists these corrections.

**Additional Correction:** Enter this additional correction from Table A 4 located at the front of the *Almanac* when obtaining a sight under non-standard atmospheric temperature and pressure conditions. This correction is a function of atmospheric pressure, temperature, and apparent altitude.

**Horizontal Parallax Correction:** This correction is unique to reducing moon sights. Obtain the H.P. correction value from the daily pages of the *Almanac*. Enter the H.P. correction table at the back of the *Almanac* with this value. The H.P. correction is a function of the limb of the moon used (upper or lower), the apparent altitude, and the H.P. correction factor. The H.P. correction is always added to the apparent altitude.

**Moon Upper Limb Correction:** Enter -30' for this correction if the sight was of the upper limb of the moon.

**Correction to Apparent Altitude:** Sum the altitude correction, the Mars or Venus additional correction, the additional correction, the horizontal parallax correction, and the moon's upper limb correction. Be careful to determine and carry the algebraic sign of the corrections and their sum correctly. Enter this sum as the correction to the apparent altitude.

**Observed Altitude:** Apply the Correction to Apparent Altitude algebraically to the apparent altitude. The result is the observed altitude.

**SECTION TWO** determines the Greenwich Mean Time (GMT) and GMT date of the sight.

**Date:** Enter the local time zone date of the sight.

**DR Latitude:** Enter the dead reckoning latitude of the vessel.

**DR Longitude:** Enter the dead reckoning longitude of the vessel.

**Observation Time:** Enter the local time of the sight as recorded on the ship's chronometer or other timepiece.

**Watch Error:** Enter a correction for any known watch error.

**Zone Time:** Correct the observation time with watch error to determine zone time.

**Zone Description:** Enter the zone description of the time zone indicated by the DR longitude. If the longitude is west of the Greenwich Meridian, the zone description is positive. Conversely, if the longitude is east of the Greenwich Meridian, the zone description is negative. The zone description represents the correction necessary to convert local time to Greenwich Mean Time.

**Greenwich Mean Time:** Add to the zone description the zone time to determine Greenwich Mean Time.

**Date:** Carefully evaluate the time correction applied above and determine if the correction has changed the date. Enter the GMT date.

**SECTION THREE** determines two of the three arguments required to enter the *Sight Reduction Tables*: Local Hour Angle (LHA) and Declination. This section employs the principle that a celestial body's LHA is the algebraic sum of its Greenwich Hour Angle (GHA) and the observer's lon-

<b><u>SECTION ONE: OBSERVED ALTITUDE</u></b>		
Body	.....	.....
Index Correction	.....	.....
Dip (height of eye)	.....	.....
Sum	.....	.....
Sextant Altitude ( $h_s$ )	.....	.....
Apparent Altitude ( $h_a$ )	.....	.....
Altitude Correction	.....	.....
Mars or Venus Additional Correction	.....	.....
Additional Correction	.....	.....
Horizontal Parallax Correction	.....	.....
Moon Upper Limb Correction	.....	.....
Correction to Apparent Altitude ( $h_a$ )	.....	.....
Observed Altitude ( $h_o$ )	.....	.....
<b><u>SECTION TWO: GMT TIME AND DATE</u></b>		
Date	.....	.....
DR Latitude	.....	.....
DR Longitude	.....	.....
Observation Time	.....	.....
Watch Error	.....	.....
Zone Time	.....	.....
Zone Description	.....	.....
Greenwich Mean Time	.....	.....
Date GMT	.....	.....
<b><u>SECTION THREE: LOCAL HOUR ANGLE AND DECLINATION</u></b>		
Tabulated GHA and $v$ Correction Factor	.....	.....
GHA Increment	.....	.....
Sidereal Hour Angle (SHA) or $v$ Correction	.....	.....
GHA	.....	.....
+ or - 360° if needed	.....	.....
Assumed Longitude (-W, +E)	.....	.....
Local Hour Angle (LHA)	.....	.....
Tabulated Declination and $d$ Correction Factor	.....	.....
$d$ Correction	.....	.....
True Declination	.....	.....
Assumed Latitude	.....	.....
<b><u>SECTION FOUR: ALTITUDE INTERCEPT AND AZIMUTH</u></b>		
Declination Increment and $d$ Interpolation Factor	.....	.....
Computed Altitude (Tabulated)	.....	.....
Double Second Difference Correction	.....	.....
Total Correction	.....	.....
Computed Altitude ( $h_c$ )	.....	.....
Observed Altitude ( $h_o$ )	.....	.....
Altitude Intercept	.....	.....
Azimuth Angle	.....	.....
True Azimuth	.....	.....

Figure 2004. Complete sight reduction form.

gitude. Therefore, the basic method employed in this section is: (1) Determine the body's GHA; (2) Determine an assumed longitude; (3) Algebraically combine the two quantities, remembering to subtract a western assumed longitude from GHA and to add an eastern longitude to GHA; and (4) Extract the declination of the body from the appropriate Almanac table, correcting the tabular value if required.

**(1) Tabulated GHA and (2)  $\nu$  Correction Factor:**

(1) For the sun, the moon, or a planet, extract the value for the whole hour of GHA corresponding to the sight. For example, if the sight was obtained at 13-50-45 GMT, extract the GHA value for 1300. For a star sight reduction, extract the value of the GHA of Aries (GHA  $\Upsilon^{\circ}$ ), again using the value corresponding to the whole hour of the time of the sight.

(2) For a planet or moon sight reduction, enter the  $\nu$  correction value. This quantity is not applicable to a sun or star sight. The  $\nu$  correction for a planet sight is found at the bottom of the column for each particular planet. The  $\nu$  correction factor for the moon is located directly beside the tabulated hourly GHA values. The  $\nu$  correction factor for the moon is always positive. If a planet's  $\nu$  correction factor is listed without sign, it is positive. If listed with a negative sign, the planet's  $\nu$  correction factor is negative. This  $\nu$  correction factor is not the magnitude of the  $\nu$  correction; it is used later to enter the Increments and Corrections table to determine the magnitude of the correction.

**GHA Increment:** The GHA increment serves as an interpolation factor, correcting for the time that the sight differed from the whole hour. For example, in the sight at 13-50-45 discussed above, this increment correction accounts for the 50 minutes and 45 seconds after the whole hour at which the sight was taken. Obtain this correction value from the Increments and Corrections tables in the *Almanac*. The entering arguments for these tables are the minutes and seconds after the hour at which the sight was taken and the body sighted. Extract the proper correction from the applicable table and enter the correction here.

**Sidereal Hour Angle or  $\nu$  Correction:** If reducing a star sight, enter the star's Sidereal Hour Angle (SHA). The SHA is found in the star column of the daily pages of the *Almanac*. The SHA combined with the GHA of Aries results in the star's GHA. The SHA entry is applicable only to a star. If reducing a planet or moon sight, obtain the  $\nu$  correction from the Increments and Corrections Table. The correction is a function of only the  $\nu$  correction factor; its magnitude is the same for both the moon and the planets.

**GHA:** A star's GHA equals the sum of the Tabulated GHA of Aries, the GHA Increment, and the star's SHA. The sun's GHA equals the sum of the Tabulated GHA and the GHA Increment. The GHA of the moon or a planet equals the sum of the Tabulated GHA, the GHA Increment, and the  $\nu$  correction.

**+ or - 360° (if needed):** Since the LHA will be determined from subtracting or adding the assumed longitude to the GHA, adjust the GHA by 360° if needed to facilitate the

addition or subtraction.

**Assumed Longitude:** If the vessel is west of the prime meridian, the assumed longitude will be subtracted from the GHA to determine LHA. If the vessel is east of the prime meridian, the assumed longitude will be added to the GHA to determine the LHA. Select the assumed longitude to meet the following two criteria: (1) When added or subtracted (as applicable) to the GHA determined above, a whole degree of LHA will result; and (2) It is the longitude closest to that DR longitude that meets criterion (1) above.

**Local Hour Angle (LHA):** Combine the body's GHA with the assumed longitude as discussed above to determine the body's LHA.

**(1) Tabulated Declination and  $d$  Correction factor:**

(1) Obtain the tabulated declination for the sun, the moon, the stars, or the planets from the daily pages of the *Almanac*. The declination values for the stars are given for the entire three day period covered by the daily page of the *Almanac*. The values for the sun, moon, and planets are listed in hourly increments. For these bodies, enter the declination value for the whole hour of the sight. For example, if the sight is at 12-58-40, enter the tabulated declination for 1200. (2) There is no  $d$  correction factor for a star sight. There are  $d$  correction factors for sun, moon, and planet sights. Similar to the  $\nu$  correction factor discussed above, the  $d$  correction factor does not equal the magnitude of the  $d$  correction; it provides the argument to enter the Increments and Corrections tables in the *Almanac*. The sign of the  $d$  correction factor, which determines the sign of the  $d$  correction, is determined by the trend of declination values, *not* the trend of  $d$  values. The  $d$  correction factor is simply an interpolation factor; therefore, to determine its sign, look at the declination values for the hours that frame the time of the sight. For example, suppose the sight was taken on a certain date at 12-30-00. Compare the declination value for 1200 and 1300 and determine if the declination has increased or decreased. If it has increased, the  $d$  correction factor is positive. If it has decreased, the  $d$  correction factor is negative.

**$d$  correction:** Enter the Increments and Corrections table with the  $d$  correction factor discussed above. Extract the proper correction, being careful to retain the proper sign.

**True Declination:** Combine the tabulated declination and the  $d$  correction to obtain the true declination.

**Assumed Latitude:** Choose as the assumed latitude that whole value of latitude closest to the vessel's DR latitude. If the assumed latitude and declination are both north or both south, label the assumed latitude *same*. If one is north and the other is south, label the assumed latitude *contrary*.

**SECTION FOUR** uses the arguments of assumed latitude, LHA, and declination determined in Section Three to enter the *Sight Reduction Tables* to determine azimuth and computed altitude. Then, Section Four compares computed and observed altitudes to calculate the altitude intercept. The navigator then has enough information to plot the line of position.

**(1) Declination Increment and (2) *d* Interpolation Factor:** Note that two of the three arguments used to enter the *Sight Reduction Tables*, LHA and latitude, are whole degree values. Section Three does not determine the third argument, declination, as a whole degree. Therefore, the navigator must interpolate in the *Sight Reduction Tables* for declination, given whole degrees of LHA and latitude. The first steps of Section Four involve this interpolation for declination. Since declination values are tabulated every whole degree in the *Sight Reduction Tables*, the declination increment is the minutes and tenths of the true declination. For example, if the true declination is  $13^{\circ} 15.6'$ , then the declination increment is  $15.6'$ . (2) The *Sight Reduction Tables* also list a *d* Interpolation Factor. This is the magnitude of the difference between the two successive tabulated values for declination that frame the true declination. Therefore, for the hypothetical declination listed above, the tabulated *d* interpolation factor listed in the table would be the difference between declination values given for  $13^{\circ}$  and  $14^{\circ}$ . If the declination increases between these two values, *d* is positive. If the declination decreases between these two values, *d* is negative.

**Computed Altitude (Tabulated):** Enter the *Sight Reduction Tables* with the following arguments: (1) LHA from Section Three; (2) assumed latitude from Section Three; (3) the whole degree value of the true declination. For example, if the true declination were  $13^{\circ} 15.6'$ , then enter the *Sight Reduction Tables* with  $13^{\circ}$  as the value for declination. Record the tabulated computed altitude.

**Double Second Difference Correction:** Use this correction when linear interpolation of declination for computed altitude is not sufficiently accurate due to the non linear change in the computed altitude as a function of declination. The need for double second difference interpolation is indicated by the *d* interpolation factor appearing in italic type followed by a small dot. When this procedure must be em-

ployed, refer to detailed instructions in the *Sight Reduction Tables* introduction.

**Total Correction:** The total correction is the sum of the double second difference (if required) and the interpolation corrections. Calculate the interpolation correction by dividing the declination increment by  $60'$  and multiply the resulting quotient by the *d* interpolation factor.

**Computed Altitude ( $h_c$ ):** Apply the total correction, being careful to carry the correct sign, to the tabulated computed altitude. This yields the computed altitude.

**Observed Altitude ( $h_o$ ):** Enter the observed altitude from Section One.

**Altitude Intercept:** Compare  $h_c$  and  $h_o$ . Subtract the smaller from the larger. The resulting difference is the magnitude of the altitude intercept. If  $h_o$  is greater than  $h_c$ , then label the altitude intercept *toward*. If  $h_c$  is greater than  $h_o$ , then label the altitude intercept *away*.

**Azimuth Angle:** Obtain the azimuth angle (*Z*) from the *Sight Reduction Tables*, using the same arguments which determined tabulated computed altitude. Visual interpolation is sufficiently accurate.

**True Azimuth:** Calculate the true azimuth ( $Z_n$ ) from the azimuth angle (*Z*) as follows:

a) If in northern latitudes:

$$\text{LHA} > 180^{\circ}, \text{ then } Z_n = Z$$

$$\text{LHA} < 180^{\circ}, \text{ then } Z_n = 360^{\circ} - Z$$

b) If in southern latitudes:

$$\text{LHA} > 180^{\circ}, \text{ then } Z_n = 180^{\circ} - Z$$

$$\text{LHA} < 180^{\circ}, \text{ then } Z_n = 180^{\circ} + Z$$

## SIGHT REDUCTION

The section above discussed the basic theory of sight reduction and proposed a method to be followed when reducing sights. This section puts that method into practice in reducing sights of a star, the sun, the moon, and planets.

### 2005. Reducing Star Sights To A Fix

On May 16, 1995, at the times indicated, the navigator takes and records the following sights:

Star	Sextant Altitude	Zone Time
Kochab	$47^{\circ} 19.1'$	20-07-43
Spica	$32^{\circ} 34.8'$	20-11-26

Height of eye is 48 feet and index correction (IC) is  $+2.1'$ . The DR latitude for both sights is  $39^{\circ}$  N. The DR longitude for the Spica sight is  $157^{\circ} 10'W$ . The DR longitude

for the Kochab sight is  $157^{\circ} 08.0'W$ . Determine the intercept and azimuth for both sights. See Figure 2005.

First, convert the sextant altitudes to observed altitudes. Reduce the Spica sight first:

Body	Spica
Index Correction	$+2.1'$
Dip (height 48 ft)	$-6.7'$
Sum	$-4.6'$
Sextant Altitude ( $h_s$ )	$32^{\circ} 34.8'$
Apparent Altitude ( $h_a$ )	$32^{\circ} 30.2'$
Altitude Correction	$-1.5'$
Additional Correction	0
Horizontal Parallax	0
Correction to $h_a$	$-1.5'$
Observed Altitude ( $h_o$ )	$32^{\circ} 28.7'$

Determine the sum of the index correction and the dip

correction. Go to the inside front cover of the *Nautical Almanac* to the table entitled DIP. This table lists dip corrections as a function of height of eye measured in either feet or meters. In the above problem, the observer's height of eye is 48 feet. The heights of eye are tabulated in intervals, with the correction corresponding to each interval listed between the interval's endpoints. In this case, 48 feet lies between the tabulated 46.9 to 48.4 feet interval; the corresponding correction for this interval is -6.7'. Add the IC and the dip correction, being careful to carry the correct sign. The sum of the corrections here is -4.6'. Apply this correction to the sextant altitude to obtain the apparent altitude ( $h_a$ ).

Next, apply the altitude correction. Find the altitude correction table on the inside front cover of the *Nautical Almanac* next to the dip table. The altitude correction varies as a function of both the type of body sighted (sun, star, or planet) and the body's apparent altitude. For the problem above, enter the star altitude correction table. Again, the correction is given within an altitude interval;  $h_a$  in this case was  $32^\circ 30.2'$ . This value lies between the tabulated endpoints  $32^\circ 00.0'$  and  $33^\circ 45.0'$ . The correction corresponding to this interval is -1.5'. Applying this correction to  $h_a$  yields an observed altitude of  $32^\circ 28.7'$ .

Having calculated the observed altitude, determine the time and date of the sight in Greenwich Mean Time:

Date	16 May 1995
DR Latitude	$39^\circ$ N
DR Longitude	$157^\circ 10'$ W
Observation Time	20-11-26
Watch Error	0
Zone Time	20-11-26
Zone Description	+10
GMT	06-11-26
GMT Date	17 May 1995

Record the observation time and then apply any watch error to determine zone time. Then, use the DR longitude at the time of the sight to determine time zone description. In this case, the DR longitude indicates a zone description of +10 hours. Add the zone description to the zone time to obtain GMT. It is important to carry the correct date when applying this correction. In this case, the +10 correction made it 06-11-26 GMT on May 17, when the date in the local time zone was May 16.

After calculating both the observed altitude and the GMT time, enter the daily pages of the *Nautical Almanac* to calculate the star's Greenwich Hour Angle (GHA) and declination.

Tab GHA $\Upsilon$	$324^\circ 28.4'$
GHA Increment	$2^\circ 52.0'$
SHA	$158^\circ 45.3'$
GHA	$486^\circ 05.7'$
+/- $360^\circ$	not required

Assumed Longitude	$157^\circ 05.7'$
LHA	$329^\circ$
Tabulated Dec/d	S $11^\circ 08.4'/n.a.$

d Correction	—
True Declination	S $11^\circ 08.4'$
Assumed Latitude	N $39^\circ$ contrary

First, record the GHA of Aries from the May 17, 1995 daily page:  $324^\circ 28.4'$ .

Next, determine the incremental addition for the minutes and seconds after 0600 from the Increments and Corrections table in the back of the *Nautical Almanac*. The increment for 11 minutes and 26 seconds is  $2^\circ 52'$ .

Then, calculate the GHA of the star. Remember:

$$\text{GHA (star)} = \text{GHA } \Upsilon + \text{SHA (star)}$$

The *Nautical Almanac* lists the SHA of selected stars on each daily page. The SHA of Spica on May 17, 1995:  $158^\circ 45.3'$ .

The *Sight Reduction Tables'* entering arguments are whole degrees of LHA and assumed latitude. Remember that  $\text{LHA} = \text{GHA} - \text{west longitude}$  or  $\text{GHA} + \text{east longitude}$ . Since in this example the vessel is in west longitude, subtract its assumed longitude from the GHA of the body to obtain the LHA. Assume a longitude meeting the criteria listed in section 2004.

From those criteria, the assumed longitude must end in 05.7 minutes so that, when subtracted from the calculated GHA, a whole degree of LHA will result. Since the DR longitude was  $157^\circ 10.0'$ , then the assumed longitude ending in 05.7' closest to the DR longitude is  $157^\circ 05.7'$ . Subtracting this assumed longitude from the calculated GHA of the star yields an LHA of  $329^\circ$ .

The next value of concern is the star's true declination. This value is found on the May 17th daily page next to the star's SHA. Spica's declination is S  $11^\circ 08.4'$ . There is no d correction for a star sight, so the star's true declination equals its tabulated declination. The assumed latitude is determined from the whole degree of latitude closest to the DR latitude at the time of the sight. In this case, the assumed latitude is N  $39^\circ$ . It is marked "contrary" because the DR latitude is north while the star's declination is south.

The following information is known: (1) the assumed position's LHA ( $329^\circ$ ) and assumed latitude ( $39^\circ$ N contrary name); and (2) the body's declination (S  $11^\circ 08.4'$ ).

Find the page in the *Sight Reduction Table* corresponding to an LHA of  $329^\circ$  and an assumed latitude of N  $39^\circ$ , with latitude contrary to declination. Enter this table with the body's whole degree of declination. In this case, the body's whole degree of declination is  $11^\circ$ . This declination corresponds to a tabulated altitude of  $32^\circ 15.9'$ . This value is for a declination of  $11^\circ$ ; the true declination is  $11^\circ 08.4'$ . Therefore, interpolate to determine the correction to add to the tabulated altitude to obtain the computed altitude.

The difference between the tabulated altitudes for  $11^\circ$  and  $12^\circ$  is given in the *Sight Reduction Tables* as the value

d; in this case,  $d = -53.0$ . Express as a ratio the declination increment (in this case,  $8.4'$ ) and the total interval between the tabulated declination values (in this case,  $60'$ ) to obtain the percentage of the distance between the tabulated declination values represented by the declination increment. Next, multiply that percentage by the increment between the two values for computed altitude. In this case:

$$\frac{8.4}{60} \times (-53.0) = -7.4$$

Subtract  $7.4'$  from the tabulated altitude to obtain the final computed altitude:  $H_c = 32^\circ 08.5'$ .

Dec Inc / + or - d	8.4' / -53.0
$h_c$ (tabulated)	$32^\circ 15.9'$
Correction (+ or -)	-7.4'
$h_c$ (computed)	$32^\circ 08.5'$

It will be valuable here to review exactly what  $h_o$  and  $h_c$  represent. Recall the methodology of the altitude-intercept method. The navigator first measures and corrects an altitude for a celestial body. This corrected altitude,  $h_o$ , corresponds to a circle of equal altitude passing through the navigator's actual position whose center is the geographic position (GP) of the body. The navigator then determines an assumed position (AP) near, but not coincident with, his actual position; he then calculates an altitude for an observer at that assumed position (AP). The circle of equal altitude passing through this assumed position is concentric with the circle of equal altitude passing through the navigator's actual position. The difference between the body's altitude at the assumed position ( $h_c$ ) and the body's observed altitude ( $h_o$ ) is equal to the differences in radii length of the two corresponding circles of equal altitude. In the above problem, therefore, the navigator knows that the equal altitude circle passing through his actual position is:

$$h_o = 32^\circ 28.7'$$

$$-h_c = \frac{32^\circ 08.5'}{20.2 \text{ NM}}$$

away from the equal altitude circle passing through his assumed position. Since  $h_o$  is greater than  $h_c$ , the navigator knows that the radius of the equal altitude circle passing through his actual position is less than the radius of the equal altitude circle passing through the assumed position. The only remaining question is: in what direction from the assumed and actual position is the body's geographic position. The *Sight Reduction Tables* also provide this final piece of information. This is the value for  $Z$  tabulated with the  $h_c$  and  $d$  values dis-

cussed above. In this case, enter the *Sight Reduction Tables* as before, with LHA, assumed latitude, and declination. Visual interpolation is sufficient. Extract the value  $Z = 143.3^\circ$ . The relation between  $Z$  and  $Z_n$ , the true azimuth, is as follows:

In northern latitudes:

$$\text{LHA} > 180^\circ, \text{ then } Z_n = Z$$

$$\text{LHA} < 180^\circ, \text{ then } Z_n = 360^\circ - Z$$

In southern latitudes:

$$\text{LHA} > 180^\circ, \text{ then } Z_n = 180^\circ - Z$$

$$\text{LHA} < 180^\circ, \text{ then } Z_n = 180^\circ + Z$$

In this case,  $\text{LHA} > 180^\circ$  and the vessel is in northern latitude. Therefore,  $Z_n = Z = 143.3^\circ \text{T}$ . The navigator now has enough information to plot a line of position.

The values for the reduction of the Kochab sight follow:

Body	Kochab
Index Correction	+2.1'
Dip Correction	-6.7'
Sum	-4.6'
$h_s$	$47^\circ 19.1'$
$h_a$	$47^\circ 14.5'$
Altitude Correction	-.9'
Additional Correction	not applicable
Horizontal Parallax	not applicable
Correction to $h_a$	-9'
$h_o$	$47^\circ 13.6'$
Date	16 May 1995
DR latitude	$39^\circ \text{N}$
DR longitude	$157^\circ 08.0' \text{W}$
Observation Time	20-07-43
Watch Error	0
Zone Time	20-07-43
Zone Description	+10
GMT	06-07-43
GMT Date	17 May 1995
Tab GHA $\sphericalangle$	$324^\circ 28.4'$
GHA Increment	$1^\circ 56.1'$
SHA	$137^\circ 18.5'$
GHA	$463^\circ 43.0'$
+/- $360^\circ$	not applicable
Assumed Longitude	$156^\circ 43.0'$
LHA	$307^\circ$
Tab Dec / $d$	$\text{N}74^\circ 10.6' / \text{n.a.}$
$d$ Correction	not applicable
True Declination	$\text{N}74^\circ 10.6'$
Assumed Latitude	$39^\circ \text{N}$ (same)
Dec Inc / + or - $d$	$10.6' / -24.8$
$h_c$	$47^\circ 12.6'$
Total Correction	-4.2'





$h_c$ (computed)	47° 08.2'
$h_o$	47° 13.6'
$a$ (intercept)	5.4 towards
$Z$	018.9°
$Z_n$	018.9°

### 2006. Reducing A Sun Sight

The example below points out the similarities between reducing a sun sight and reducing a star sight. It also demonstrates the additional corrections required for low altitude (<10°) sights and sights taken during non-standard temperature and pressure conditions.

On June 16, 1994, at 05-15-23 local time, at DR position L 30°N  $\lambda$  45°W, a navigator takes a sight of the sun's upper limb. The navigator has a height of eye of 18 feet, the temperature is 88° F, and the atmospheric pressure is 982 mb. The sextant altitude is 3° 20.2'. There is no index error. Determine the observed altitude. See Figure 2007.

Body	Sun UL
Index Correction	0
Dip Correction (18 ft)	-4.1'
Sum	-4.1'
$h_s$	3° 20.2'
$h_a$	3° 16.1'
Altitude Correction	-29.4'
Additional Correction	+1.4'
Horizontal Parallax	0
Correction to $h_a$	-28.0'
$h_o$	2° 48.1'

Apply the index and dip corrections to  $h_s$  to obtain  $h_a$ . Because  $h_a$  is less than 10°, use the special altitude correction table for sights between 0° and 10° located on the right inside front page of the *Nautical Almanac*.

Enter the table with the apparent altitude, the limb of the sun used for the sight, and the period of the year. Interpolation for the apparent altitude is not required. In this case, the table yields a correction of -29.4'. The correction's algebraic sign is found at the head of each group of entries and at every change of sign.

The additional correction is required because of the non-standard temperature and atmospheric pressure under which the sight was taken. The correction for these non-standard conditions is found in the *Additional Corrections* table located on page A4 in the front of the *Nautical Almanac*.

First, enter the *Additional Corrections* table with the temperature and pressure to determine the correct zone letter: in this case, zone L. Then, locate the correction in the L column corresponding to the apparent altitude of 3° 16.1'. Interpolate between the table arguments of 3° 00.0' and 3° 30.0' to determine the additional correction: +1.4'. The total correction to the apparent altitude is the sum of the altitude and additional corrections: -28.0'. This results in an  $h_o$  of 2° 48.1'.

Next, determine the sun's GHA and declination. Again, this process is similar to the star sights reduced above. Notice, however, that SHA, a quantity unique to star sight reduction, is not used in sun sight reduction.

Date	June 16, 1994
DR Latitude	N30° 00.0'
DR Longitude	W045° 00.0'
Observation Time	05-15-23
Watch Error	0
Zone Time	05-15-23
Zone Description	+03
GMT	08-15-23
Date GMT	June 16, 1994
Tab GHA / $v$	299° 51.3' / n.a.
GHA Increment	3° 50.8'
SHA or $v$ correction	not applicable
GHA	303° 42.1'
Assumed Longitude	44° 42.1' W
LHA	259°
Tab Declination / $d$	N23° 20.5' / +0.1'
$d$ Correction	0.0
True Declination	N23° 20.5'
Assumed Latitude	N30° (same)

Determining the sun's GHA is less complicated than determining a star's GHA. The *Nautical Almanac's* daily pages list the sun's GHA in hourly increments. In this case, the sun's GHA at 0800 GMT on June 16, 1994 is 299° 51.3'. The  $v$  correction is not applicable for a sun sight; therefore, applying the increment correction yields the sun's GHA. In this case, the GHA is 303° 42.1'.

Determining the sun's LHA is similar to determining a star's LHA. In determining the sun's declination, however, an additional correction not encountered in the star sight, the  $d$  correction, must be considered. The bottom of the sun column on the daily pages of the *Nautical Almanac* lists the  $d$  value. This is an interpolation factor for the sun's declination. The sign of the  $d$  factor is not given; it must be determined by noting from the *Almanac* if the sun's declination is increasing or decreasing throughout the day. If it is increasing, the factor is positive; if it is decreasing, the factor is negative. In the above problem, the sun's declination is increasing throughout the day. Therefore, the  $d$  factor is +0.1.

Having obtained the  $d$  factor, enter the 15 minute increment and correction table. Under the column labeled " $v$  or  $d$  corr<sup>n</sup>," find the value for  $d$  in the left hand column. The corresponding number in the right hand column is the correction; apply it to the tabulated declination. In this case, the correction corresponding to a  $d$  value of +0.1 is 0.0'.

The final step will be to determine  $h_c$  and  $Z_n$ . Enter the *Sight Reduction Tables* with an LHA of 259°, a declination of N23° 20.5', and an assumed latitude of 30°N.

Declination Increment / + or - $d$	20.5' / +31.5
Tabulated Altitude	2° 28.8'



Correction (+ or -)	+10.8'
Computed Altitude ( $h_c$ )	2° 39.6'
Observed Altitude ( $h_o$ )	2° 48.1'
Intercept	8.5 NM (towards)
Z	064.7°
Z <sub>n</sub>	064.7°

### 2007. Reducing A Moon Sight

The moon is easy to identify and is often visible during the day. However, the moon's proximity to the earth requires applying additional corrections to  $h_a$  to obtain  $h_o$ . This section will cover moon sight reduction.

At 10-00-00 GMT, June 16, 1994, the navigator obtains a sight of the moon's upper limb.  $H_s$  is 26° 06.7'. Height of eye is 18 feet; there is no index error. Determine  $h_o$ , the moon's GHA, and the moon's declination. See Figure 2007.

Body	Moon (UL)
Index Correction	0.0'
Dip (18 feet)	-4.1'
Sum	-4.1'
Sextant Altitude ( $h_s$ )	26° 06.7'
Apparent Altitude ( $h_a$ )	26° 02.6'
Altitude Correction	+60.5'
Additional Correction	0.0'
Horizontal Parallax (58.4)	+4.0'
Moon Upper Limb Correction	-30.0'
Correction to $h_a$	+34.5'
Observed Altitude ( $h_o$ )	26° 37.1'

This procedure demonstrates the extra corrections required for obtaining  $h_o$  for a moon sight. Apply the index and dip corrections and in the same manner as for star and sun sights. The altitude correction comes from tables located on the inside back covers of the *Nautical Almanac*.

In this case, the apparent altitude was 26° 02.6'. Enter the altitude correction table for the moon with the above apparent altitude. Interpolation is not required. The correction is +60.5'. The additional correction in this case is not applicable because the sight was taken under standard temperature and pressure conditions.

The horizontal parallax correction is unique to moon sights. The table for determining this HP correction is on the back inside cover of the *Nautical Almanac*. First, go to the daily page for June 16 at 10-00-00 GMT. In the column for the moon, find the HP correction factor corresponding to 10-00-00. Its value is 58.4. Take this value to the HP correction table on the inside back cover of the *Almanac*. Notice that the HP correction columns line up vertically with the moon altitude correction table columns. Find the HP correction column directly under the altitude correction table heading corresponding to the apparent altitude. Enter that column with the HP correction factor from the daily pages. The column has two sets of figures listed under "U" and "L" for upper and lower limb, respectively. In this case, trace down the "U" column until it intersects with the HP correction fac-

tor of 58.4. Interpolating between 58.2 and 58.5 yields a value of +4.0' for the horizontal parallax correction.

The final correction is a constant -30.0' correction to  $h_a$  applied only to sights of the moon's upper limb. This correction is always negative; apply it only to sights of the moon's upper limb, not its lower limb. The total correction to  $h_a$  is the sum of all the corrections; in this case, this total correction is +34.5 minutes.

To obtain the moon's GHA, enter the daily pages in the moon column and extract the applicable data just as for a star or sun sight. Determining the moon's GHA requires an additional correction, the  $v$  correction.

GHA moon and $v$	245° 45.1' and +11.3
GHA Increment	0° 00.0'
$v$ Correction	+0.1'
GHA	245° 45.2'

First, record the GHA of the moon for 10-00-00 on June 16, 1994, from the daily pages of the *Nautical Almanac*. Record also the  $v$  correction factor; in this case, it is +11.3. The  $v$  correction factor for the moon is always positive. The increment correction is, in this case, zero because the sight was recorded on the even hour. To obtain the  $v$  correction, go to the tables of increments and corrections. In the 0 minute table in the  $v$  or  $d$  correction columns, find the correction that corresponds to a  $v = 11.3$ . The table yields a correction of +0.1'. Adding this correction to the tabulated GHA gives the final GHA as 245° 45.2'.

Finding the moon's declination is similar to finding the declination for the sun or stars. Go to the daily pages for June 16, 1994; extract the moon's declination and  $d$  factor.

Tabulated Declination / $d$	S 00° 13.7' / +12.1
$d$ Correction	+0.1'
True Declination	S 00° 13.8'

The tabulated declination and the  $d$  factor come from the *Nautical Almanac's* daily pages. Record the declination and  $d$  correction and go to the increment and correction pages to extract the proper correction for the given  $d$  factor. In this case, go to the correction page for 0 minutes. The correction corresponding to a  $d$  factor of +12.1 is +0.1. It is important to extract the correction with the correct algebraic sign. The  $d$  correction may be positive or negative depending on whether the moon's declination is increasing or decreasing in the interval covered by the  $d$  factor. In this case, the moon's declination at 10-00-00 GMT on 16 June was S 00° 13.7'; at 11-00-00 on the same date the moon's declination was S 00° 25.8'. Therefore, since the declination was increasing over this period, the  $d$  correction is positive. Do not determine the sign of this correction by noting the trend in the  $d$  factor. In other words, had the  $d$  factor for 11-00-00 been a value less than 12.1, that would not indicate that the  $d$  correction should be negative. Remember that the  $d$  factor is analogous to an interpolation



factor; it provides a correction to declination. Therefore, the trend in declination values, not the trend in  $d$  values, controls the sign of the  $d$  correction. Combine the tabulated declination and the  $d$  correction factor to determine the true declination. In this case, the moon's true declination is  $S 00^{\circ} 13.8'$

Having obtained the moon's GHA and declination, calculate LHA and determine the assumed latitude. Enter the *Sight Reduction Table* with the LHA, assumed latitude, and calculated declination. Calculate the intercept and azimuth in the same manner used for star and sun sights.

### 2008. Reducing A Planet Sight

There are four navigational planets: Venus, Mars, Jupiter, and Saturn. Reducing a planet sight is similar to reducing a sun or star sight, but there are a few important differences. This section will cover the procedure for determining  $h_o$ , the GHA and the declination for a planet sight.

On July 27, 1995, at 09-45-20 GMT, you take a sight of Mars.  $H_s$  is  $33^{\circ} 20.5'$ . The height of eye is 25 feet, and the index correction is  $+0.2'$ . Determine  $h_o$ , GHA, and declination. See Figure 2008.

Body	Mars
Index Correction	$+0.2'$
Dip Correction (25 feet)	$-4.9'$
Sum	$-4.7'$
$h_s$	$33^{\circ} 20.5'$
$h_a$	$33^{\circ} 15.8'$
Altitude Correction	$-1.5'$
Additional Correction	Not applicable
Horizontal Parallax	Not applicable
Additional Correction for Mars	$+0.1'$
Correction to $h_a$	$-1.4'$
$h_o$	$33^{\circ} 14.4'$

The table above demonstrates the similarity between reducing planet sights and reducing sights of the sun and stars. Calculate and apply the index and dip corrections exactly as for any other sight. Take the resulting apparent altitude and enter the altitude correction table for the stars and planets on the inside front cover of the *Nautical Almanac*.

In this case, the altitude correction for  $33^{\circ} 15.8'$  results in a correction of  $-1.5'$ . The additional correction is not applicable

because the sight was taken at standard temperature and pressure; the horizontal parallax correction is not applicable to a planet sight. All that remains is the correction specific to Mars or Venus. The altitude correction table in the *Nautical Almanac* also contains this correction. Its magnitude is a function of the body sighted (Mars or Venus), the time of year, and the body's apparent altitude. Entering this table with the data for this problem yields a correction of  $+0.1'$ . Applying these corrections to  $h_a$  results in an  $h_o$  of  $33^{\circ} 14.4'$ .

Tabulated GHA / $v$	$256^{\circ} 10.6' / 1.1$
GHA Increment	$11^{\circ} 20.0'$
$v$ correction	$+0.8'$
GHA	$267^{\circ} 31.4'$

The only difference between determining the sun's GHA and a planet's GHA lies in applying the  $v$  correction. Calculate this correction from the  $v$  or  $d$  correction section of the Increments and Correction table in the *Nautical Almanac*.

Find the  $v$  factor at the bottom of the planets' GHA columns on the daily pages of the *Nautical Almanac*. For Mars on July 27, 1995, the  $v$  factor is 1.1. If no algebraic sign precedes the  $v$  factor, add the resulting correction to the tabulated GHA. Subtract the resulting correction only when a negative sign precedes the  $v$  factor. Entering the  $v$  or  $d$  correction table corresponding to 45 minutes yields a correction of  $0.8'$ . Remember, because no sign preceded the  $v$  factor on the daily pages, add this correction to the tabulated GHA. The final GHA is  $267^{\circ} 31.4'$ .

Tabulated Declination / $d$	$S 01^{\circ} 06.1' / 0.6$
$d$ Correction	$+0.5'$
True Declination	$S 01^{\circ} 06.6'$

Read the tabulated declination directly from the daily pages of the *Nautical Almanac*. The  $d$  correction factor is listed at the bottom of the planet column; in this case, the factor is 0.6. Note the trend in the declination values for the planet; if they are increasing during the day, the correction factor is positive. If the planet's declination is decreasing during the day, the correction factor is negative. Next, enter the  $v$  or  $d$  correction table corresponding to 45 minutes and extract the correction for a  $d$  factor of 0.6. The correction in this case is  $+0.5'$ .

From this point, reducing a planet sight is exactly the same as reducing a sun sight.

## MERIDIAN PASSAGE

This section covers determining both latitude and longitude at the meridian passage of the sun, or Local Apparent Noon (LAN). Determining a vessel's latitude at LAN requires calculating the sun's zenith distance and declination and combining them according to the rules discussed below.

Latitude at LAN is a special case of the navigational triangle where the sun is on the observer's meridian and the

triangle becomes a straight north/south line. No "solution" is necessary, except to combine the sun's zenith distance and its declination according to the rules discussed below.

Longitude at LAN is a function of the time elapsed since the sun passed the Greenwich meridian. The navigator must determine the time of LAN and calculate the GHA of the sun at that time. The following examples demonstrates these processes.



### 2009. Latitude At Meridian Passage

At 1056 ZT, May 16, 1995, a vessel's DR position is L 40° 04.3'N and  $\lambda$  157° 18.5' W. The ship is on course 200°T at a speed of ten knots. (1) Calculate the first and second estimates of Local Apparent Noon. (2) The navigator actually observes LAN at 12-23-30 zone time. The sextant altitude at LAN is 69° 16.0'. The index correction is +2.1' and the height of eye is 45 feet. Determine the vessel's latitude.

Date	16 May 1995
DR Latitude (1156 ZT)	39° 55.0' N
DR Longitude (1156 ZT)	157° 23.0' W
Central Meridian	150° W
d Longitude (arc)	7° 23' W
d Longitude (time)	+29 min. 32 sec
Meridian Passage (LMT)	1156
ZT (first estimate)	12-25-32
DR Longitude (12-25-32)	157° 25.2'
d Longitude (arc)	7° 25.2'
d Longitude (time)	+29 min. 41 sec
Meridian Passage	1156
ZT (second estimate)	12-25-41
ZT (actual transit)	12-23-30 local
Zone Description	+10
GMT	22-23-30
Date (GMT)	16 May 1995
Tabulated Declination / d correction	N 19° 09.0' / +0.6
True Declination	N 19° 09.2'
Index Correction	+2.1'
Dip (48 ft)	-6.7'
Sum	-4.6'
$h_s$ (at LAN)	69° 16.0'
$h_a$	69° 11.4'
Altitude Correction	+15.6'
89° 60'	89° 60.0'
$h_o$	69° 27.0'
Zenith Distance	N 20° 33.0'
True Declination	N 19° 09.2'
Latitude	39° 42.2'

First, determine the time of meridian passage from the daily pages of the *Nautical Almanac*. In this case, the meridian passage for May 16, 1995, is 1156. That is, the sun crosses the central meridian of the time zone at 1156 ZT and the observer's local meridian at 1156 local time. Next, determine the vessel's DR longitude for the time of meridian passage. In this case, the vessel's 1156 DR longitude is 157° 23.0' W. Determine the time zone in which this DR longitude falls and record the longitude of that time zone's central meridian. In this case, the central meridian is 150° W. Enter the Conversion of Arc to Time table in the *Nautical Almanac* with the difference between the DR longitude and the central meridian longitude. The conversion for 7° of arc is 28<sup>m</sup> of time, and the conversion for 23' of arc is 1<sup>m</sup>32<sup>s</sup> of time. Sum these two times. If the DR position is west of the

central meridian (as it is in this case), add this time to the time of tabulated meridian passage. If the longitude difference is to the east of the central meridian, subtract this time from the tabulated meridian passage. In this case, the DR position is west of the central meridian. Therefore, add 29 minutes and 32 seconds to 1156, the tabulated time of meridian passage. The estimated time of LAN is 12-25-32 ZT.

This first estimate for LAN does not take into account the vessel's movement. To calculate the *second estimate* of LAN, first determine the DR longitude for the time of first estimate of LAN (12-25-32 ZT). In this case, that longitude would be 157° 25.2' W. Then, calculate the difference between the longitude of the 12-25-32 DR position and the central meridian longitude. This would be 7° 25.2'. Again, enter the arc to time conversion table and calculate the time difference corresponding to this longitude difference. The correction for 7° of arc is 28' of time, and the correction for 25.2' of arc is 1'41" of time. Finally, apply this time correction to the original tabulated time of meridian passage (1156 ZT). The resulting time, 12-25-41 ZT, is the *second estimate* of LAN.

Solving for latitude requires that the navigator calculate two quantities: the sun's declination and the sun's zenith distance. First, calculate the sun's true declination at LAN. The problem states that LAN is 12-28-30. (Determining the exact time of LAN is covered in section 2010.) Enter the time of observed LAN and add the correct zone description to determine GMT. Determine the sun's declination in the same manner as in the sight reduction problem in section 2006. In this case, the tabulated declination was N 19° 19.1', and the d correction +0.2'. The true declination, therefore, is N 19° 19.3'.

Next, calculate zenith distance. Recall from Navigational Astronomy that zenith distance is simply 90° - observed altitude. Therefore, correct  $h_s$  to obtain  $h_a$ ; then correct  $h_a$  to obtain  $h_o$ . Then, subtract  $h_o$  from 90° to determine the zenith distance. Name the zenith distance North or South depending on the relative position of the observer and the sun's declination. If the observer is to the north of the sun's declination, name the zenith distance north. Conversely, if the observer is to the south of the sun's declination, name the zenith distance south. In this case, the DR latitude is N 39° 55.0' and the sun's declination is N 19° 19.3'. The observer is to the north of the sun's declination; therefore, name the zenith distance north. Next, compare the names of the zenith distance and the declination. If their names are the same (i.e., both are north or both are south), add the two values together to obtain the latitude. This was the case in this problem. Both the sun's declination and zenith distance were north; therefore, the observer's latitude is the sum of the two.

If the name of the body's zenith distance is contrary to the name of the sun's declination, then subtract the smaller of the two quantities from the larger, carrying for the name of the difference the name of the larger of the two quantities. The result is the observer's latitude. The following examples illustrate this process.

Zenith Distance	N 25°	Zenith Distance	S 50°
<u>True Declination</u>	<u>S 15°</u>	<u>True Declination</u>	<u>N 10°</u>
Latitude	N 10°	Latitude	S 40°



### 2010. Longitude At Meridian Passage

Determining a vessel's longitude at LAN is straightforward. In the western hemisphere, the sun's GHA at LAN equals the vessel's longitude. In the eastern hemisphere, subtract the sun's GHA from  $360^\circ$  to determine longitude. The difficult part lies in determining the precise moment of meridian passage.

Determining the time of meridian passage presents a problem because the sun appears to hang for a finite time at its local maximum altitude. Therefore, noting the time of maximum sextant altitude is not sufficient for determining the precise time of LAN. Two methods are available to obtain LAN with a precision sufficient for determining longitude: (1) the graphical method and (2) the calculation method. The graphical method is discussed first below.

See Figure 2010. Approximately 30 minutes before the estimated time of LAN, measure and record sextant altitudes and their corresponding times. Continue taking sights for about 30 minutes after the sun has descended from the maximum recorded altitude. Increase the sighting frequency near the predicted meridian passage. One sight every 20-30 seconds should yield good results near meridian passage; less frequent sights are required before and after.

Plot the resulting data on a graph of sextant altitude versus time. Fair a curve through the plotted data. Next, draw a series of horizontal lines across the curve formed by the data points. These lines will intersect the faired

curve at two different points. The x coordinates of the points where these lines intersect the faired curve represent the two different times when the sun's altitude is equal (one time when the sun was ascending; the other time when the sun was descending). Draw three such lines, and ensure the lines have sufficient vertical separation. For each line, average the two times where it intersects the faired curve. Finally, average the three resulting times to obtain a final value for the time of LAN. From the *Nautical Almanac*, determine the sun's GHA at that time; this is your longitude in the western hemisphere. In the eastern hemisphere, subtract the sun's GHA from  $360^\circ$  to determine longitude.

The second method of determining LAN is similar to the first. Estimate the time of LAN as discussed above. Measure and record the sun's altitude as the sun approaches its maximum altitude. As the sun begins to descend, set the sextant to correspond to the altitude recorded just before the sun's reaching its maximum altitude. Note the time when the sun is again at that altitude. Average the two times. Repeat this procedure with two other altitudes recorded before LAN, each time pre-setting the sextant to those altitudes and recording the corresponding times that the sun, now on its descent, passes through those altitudes. Average these corresponding times. Take a final average among the three averaged times; the result will be the time of meridian passage. Determine the vessel's longitude by determining the sun's GHA at the exact time of LAN.

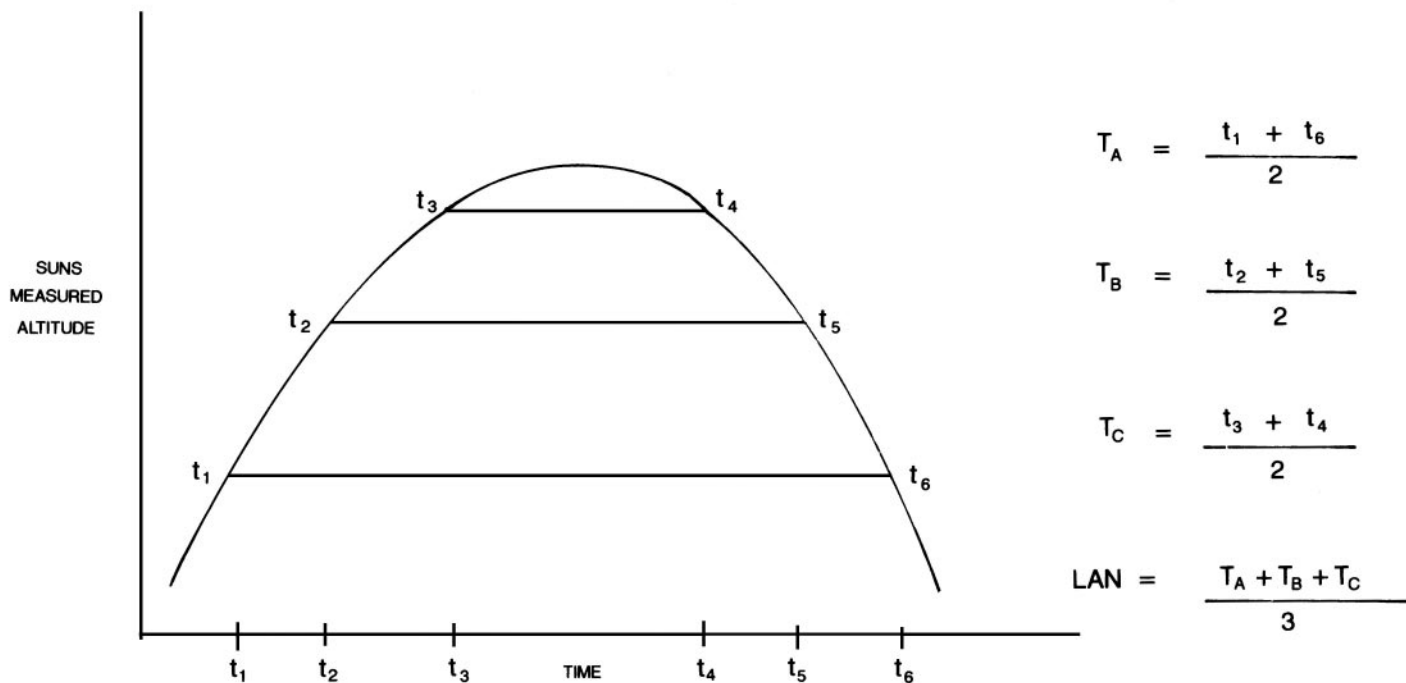


Figure 2010. Time of LAN.

## LATITUDE BY POLARIS

### 2011. Latitude By Polaris

Since Polaris is always within about  $1^\circ$  of the North Pole, the altitude of Polaris, with a few minor corrections, equals the latitude of the observer. This relationship makes Polaris an extremely important navigational star in the northern hemisphere.

The corrections are necessary because Polaris orbits in a small circle around the pole. When Polaris is at the exact same altitude as the pole, the correction is zero. At two points in its orbit it is in a direct line with the observer and the pole, either nearer than or beyond the pole. At these points the corrections are maximum. The following example illustrates converting a Polaris sight to latitude.

At 23-18-56 GMT, on April 21, 1994, at DR  $\lambda=37^\circ 14.0'$  W,  $L = 50^\circ 23.8'$  N, the observed altitude of Polaris ( $h_o$ ) is  $49^\circ 31.6'$ . Find the vessel's latitude.

To solve this problem, use the equation:

$$\text{Latitude} = h_o - 1^\circ + A_0 + A_1 + A_2$$

where  $h_o$  is the sextant altitude ( $h_s$ ) corrected as in any other star sight;  $1^\circ$  is a constant; and  $A_0$ ,  $A_1$ , and  $A_2$  are correction factors from the Polaris tables found in the *Nautical Almanac*. These three correction factors are always positive. One needs the following information to enter the tables: LHA of Aries, DR latitude, and the month of the year. Therefore:

Tabulated GHA $\Upsilon$ (2300 hrs.)	$194^\circ 32.7'$
Increment (18-56)	$4^\circ 44.8'$
GHA $\Upsilon$	$199^\circ 17.5'$
DR Longitude (-W +E)	$37^\circ 14.0'$

LHA $\Upsilon$	$162^\circ 03.5'$
$A_0$ ( $162^\circ 03.5'$ )	$+1^\circ 25.4'$
$A_1$ ( $L = 50^\circ\text{N}$ )	$+0.6'$
$A_2$ (April)	$+0.9'$
Sum	$1^\circ 26.9'$
Constant	$-1^\circ 00.0'$
Observed Altitude	$49^\circ 31.6'$
Total Correction	$+26.9'$
Latitude	$N 49^\circ 58.5'$

Enter the Polaris table with the calculated LHA of Aries ( $162^\circ 03.5'$ ). See Figure 2011. The first correction,  $A_0$ , is a function solely of the LHA of Aries. Enter the table column indicating the proper range of LHA of Aries; in this case, enter the  $160^\circ$ - $169^\circ$  column. The numbers on the left hand side of the  $A_0$  correction table represent the whole degrees of LHA  $\Upsilon$ ; interpolate to determine the proper  $A_0$  correction. In this case, LHA  $\Upsilon$  was  $162^\circ 03.5'$ . The  $A_0$  correction for LHA =  $162^\circ$  is  $1^\circ 25.4'$  and the  $A_0$  correction for LHA =  $163^\circ$  is  $1^\circ 26.1'$ . The  $A_0$  correction for  $162^\circ 03.5'$  is  $1^\circ 25.4'$ .

To calculate the  $A_1$  correction, enter the  $A_1$  correction table with the DR latitude, being careful to stay in the  $160^\circ$ - $169^\circ$  LHA column. There is no need to interpolate here; simply choose the latitude that is closest to the vessel's DR latitude. In this case,  $L$  is  $50^\circ\text{N}$ . The  $A_1$  correction corresponding to an LHA range of  $160^\circ$ - $169^\circ$  and a latitude of  $50^\circ\text{N}$  is  $+0.6'$ .

Finally, to calculate the  $A_2$  correction factor, stay in the  $160^\circ$ - $169^\circ$  LHA  $\Upsilon$  column and enter the  $A_2$  correction table. Follow the column down to the month of the year; in this case, it is April. The correction for April is  $+0.9'$ .

Sum the corrections, remembering that all three are always positive. Subtract  $1^\circ$  from the sum to determine the total correction; then apply the resulting value to the observed altitude of Polaris. This is the vessel's latitude.

POLARIS (POLE STAR) TABLES, 1994  
FOR DETERMINING LATITUDE FROM SEXTANT ALTITUDE AND FOR AZIMUTH

LHA ARIES	120° - 129°	130° - 139°	140° - 149°	150° - 159°	160° - 169°	170° - 179°	180° - 189°	190° - 199°	200° - 209°	210° - 219°	220° - 229°	230° - 239°
	a <sub>0</sub>	a <sub>0</sub>	a <sub>0</sub>	a <sub>0</sub>	a <sub>0</sub>	a <sub>0</sub>	a <sub>0</sub>	a <sub>0</sub>	a <sub>0</sub>	a <sub>0</sub>	a <sub>0</sub>	a <sub>0</sub>
0	0 53.9	I 01.8	I 09.7	I 17.2	I 24.1	I 30.3	I 35.5	I 39.6	I 42.5	I 44.1	I 44.3	I 43.2
1	54.7	02.6	10.4	17.9	24.8	30.9	36.0	40.0	42.7	44.2	44.3	43.0
2	55.5	03.4	11.2	18.6	25.4	31.4	36.4	40.3	42.9	44.3	44.2	42.8
3	56.3	04.2	12.0	19.3	26.1	32.0	36.9	40.6	43.1	44.3	44.1	42.6
4	57.1	05.0	12.7	20.0	26.7	32.5	37.3	40.9	43.3	44.4	44.0	42.4
5	0 57.8	I 05.8	I 13.5	I 20.7	I 27.3	I 33.0	I 37.7	I 41.2	I 43.5	I 44.4	I 43.9	I 42.1
6	58.6	06.6	14.2	21.4	27.9	33.5	38.1	41.5	43.6	44.4	43.8	41.9
7	0 59.4	I 07.3	I 15.0	I 22.1	I 28.5	I 34.1	I 38.5	I 41.8	I 43.8	I 44.4	I 43.7	I 41.6
8	I 00.2	08.1	15.7	22.8	29.1	34.6	38.9	42.0	43.9	44.4	43.5	41.3
9	01.0	08.9	16.4	23.5	29.7	35.0	39.3	42.3	44.0	44.4	43.4	41.0
10	I 01.8	I 09.7	I 17.2	I 24.1	I 30.3	I 35.5	I 39.6	I 42.5	I 44.1	I 44.3	I 43.2	I 40.7
Lat.	a <sub>1</sub>	a <sub>1</sub>	a <sub>1</sub>	a <sub>1</sub>	a <sub>1</sub>	a <sub>1</sub>	a <sub>1</sub>	a <sub>1</sub>	a <sub>1</sub>	a <sub>1</sub>	a <sub>1</sub>	a <sub>1</sub>
0	0.2	0.2	0.3	0.3	0.4	0.4	0.5	0.6	0.6	0.6	0.6	0.6
10	.3	.3	.3	.4	.4	.5	.5	.6	.6	.6	.6	.6
20	.3	.4	.4	.4	.4	.5	.5	.6	.6	.6	.6	.6
30	.4	.4	.4	.5	.5	.5	.5	.6	.6	.6	.6	.6
40	0.5	0.5	0.5	0.5	0.5	0.6	0.6	0.6	0.6	0.6	0.6	0.6
45	.5	.5	.5	.6	.6	.6	.6	.6	.6	.6	.6	.6
50	.6	.6	.6	.6	.6	.6	.6	.6	.6	.6	.6	.6
55	.7	.7	.7	.7	.6	.6	.6	.6	.6	.6	.6	.6
60	.8	.8	.7	.7	.7	.7	.6	.6	.6	.6	.6	.6
62	0.8	0.8	0.8	0.8	0.7	0.7	0.7	0.6	0.6	0.6	0.6	0.6
64	.9	.9	.8	.8	.8	.7	.7	.6	.6	.6	.6	.6
66	0.9	0.9	.9	.8	.8	.7	.7	.6	.6	.6	.6	.6
68	1.0	1.0	0.9	0.9	0.8	0.8	0.7	0.7	0.6	0.6	0.6	0.6
Month	a <sub>2</sub>	a <sub>2</sub>	a <sub>2</sub>	a <sub>2</sub>	a <sub>2</sub>	a <sub>2</sub>	a <sub>2</sub>	a <sub>2</sub>	a <sub>2</sub>	a <sub>2</sub>	a <sub>2</sub>	a <sub>2</sub>
Jan.	0.6	0.6	0.6	0.5	0.5	0.5	0.4	0.4	0.4	0.4	0.4	0.4
Feb.	.8	.8	.7	.7	.6	.6	.5	.5	.4	.4	.4	.3
Mar.	0.9	0.9	0.9	.8	.8	.7	.6	.6	.5	.5	.4	.4
Apr.	1.0	1.0	1.0	0.9	0.9	0.8	0.8	0.7	0.7	0.6	0.5	0.5
May	0.9	1.0	1.0	1.0	1.0	0.9	.9	.9	.8	.8	.7	.6
June	.8	0.9	0.9	0.9	0.9	1.0	.9	.9	.9	.9	.8	.8
July	0.7	0.7	0.8	0.8	0.8	0.9	0.9	0.9	0.9	0.9	0.9	0.9
Aug.	.5	.5	.6	.6	.7	.7	.8	.8	.8	.9	.9	.9
Sept.	.3	.4	.4	.5	.5	.6	.6	.7	.7	.7	.8	.8
Oct.	0.3	0.3	0.3	0.3	0.3	0.4	0.4	0.5	0.5	0.6	0.6	0.7
Nov.	.2	.2	.2	.2	.2	.2	.2	.3	.3	.4	.5	.5
Dec.	0.3	0.2	0.2	0.2	0.1	0.1	0.1	0.2	0.2	0.2	0.3	0.4
Lat.	AZIMUTH											
0	359.2	359.2	359.3	359.3	359.4	359.5	359.6	359.7	359.8	0.0	0.1	0.2
20	359.2	359.2	359.2	359.3	359.4	359.5	359.6	359.7	359.8	0.0	0.1	0.3
40	359.0	359.0	359.1	359.1	359.2	359.3	359.5	359.6	359.8	0.0	0.1	0.3
50	358.8	358.8	358.9	359.0	359.1	359.2	359.4	359.6	359.8	0.0	0.2	0.4
55	358.7	358.7	358.7	358.8	359.0	359.1	359.3	359.5	359.7	0.0	0.2	0.4
60	358.5	358.5	358.6	358.7	358.8	359.0	359.2	359.5	359.7	0.0	0.2	0.5
65	358.2	358.2	358.3	358.4	358.6	358.8	359.1	359.4	359.6	359.9	0.3	0.6

Figure 2011. Excerpt from the Polaris Tables.

