

# CHAPTER 24

## THE SAILINGS

### INTRODUCTION

#### 2400. Introduction

Dead reckoning involves the determination of one's present or future position by projecting the ship's course and distance run from a known position. A closely related problem is that of finding the course and distance from one known point to another known point. For short distances, these problems are easily solved directly on charts, but for long distances, a purely mathematical solution is often a better method. Collectively, these methods are called **The Sailings**.

Navigational computer programs and calculators commonly contain algorithms for computing all of the problems of the sailings. For those situations when a calculator is not available, this chapter also discusses sailing solutions by Table 4, the Traverse Tables.

#### 2401. Rhumb Lines And Great Circles

The principal advantage of a rhumb line is that it maintains constant true direction. A ship following the rhumb line between two places does not change true course. A rhumb line makes the same angle with all meridians it crosses and appears as a straight line on a Mercator chart. For any other case, the difference between the rhumb line and the great circle connecting two points increases (1) as the latitude increases, (2) as the difference of latitude between the two points decreases, and (3) as the difference of longitude increases.

A great circle is the intersection of the surface of a sphere and a plane passing through the center of the sphere. It is the largest circle that can be drawn on the surface of the sphere, and is the shortest distance along the surface between any two points. Any two points are connected by only one great circle unless the points are **antipodal** ( $180^\circ$  apart on the earth), and then an infinite number of great circles passes through them. Every great circle bisects every other great circle. Thus, except for the equator, every great circle lies exactly half in the Northern Hemisphere and half in the Southern Hemisphere. Any two points  $180^\circ$  apart on a great circle have the same latitude numerically, but contrary names, and are  $180^\circ$  apart in longitude. The point of greatest latitude is called the **vertex**. For each great circle, there is a vertex in each hemisphere,  $180^\circ$  apart in longitude. At these points the great circle is tangent to a parallel of latitude, and its direction is due east-west. On each side

of these vertices the direction changes progressively until the intersection with the equator is reached,  $90^\circ$  in longitude away, where the great circle crosses the equator at an angle equal to the latitude of the vertex.

On a Mercator chart a great circle appears as a sine curve extending equal distances each side of the equator. The rhumb line connecting any two points of the great circle on the same side of the equator is a chord of the curve. Along any intersecting meridian the great circle crosses at a higher latitude than the rhumb line. If the two points are on opposite sides of the equator, the direction of curvature of the great circle relative to the rhumb line changes at the equator. The rhumb line and great circle may intersect each other, and if the points are equal distances on each side of the equator, the intersection takes place at the equator.

Great circle sailing takes advantage of the shorter distance along the great circle between two points, rather than the longer rhumb line. The arc of the great circle between the points is called the **great circle track**. If it could be followed exactly, the destination would be dead ahead throughout the voyage (assuming course and heading were the same). The rhumb line appears the more direct route on a Mercator chart because of chart distortion. The great circle crosses meridians at higher latitudes, where the distance between them is less. This is why the great circle route is shorter than the rhumb line.

The decision as to whether or not to use great-circle sailing depends upon the conditions. The saving in distance should be worth the additional effort, and of course the great circle route cannot cross land, nor should it carry the vessel into dangerous waters. **Composite sailing** (see section 2402 and section 2411) may save time and distance over the rhumb line track without leading the vessel into danger.

Since great circles other than a meridian or the equator are curved lines whose true direction changes continually, the navigator does not attempt to follow it exactly. Rather, he selects a number of points along the great circle, constructs rhumb lines between the points, and follows these rhumb lines from point to point.

#### 2402. Kinds Of Sailings

There are seven types of sailings:

1. **Plane sailing** solves problems involving a single course and distance, difference of latitude, and de-

parture, in which the earth is regarded as a plane surface. This method, therefore, provides solution for latitude of the point of arrival, but not for longitude. To calculate the longitude, the spherical sailings are necessary. Do not use this method for distances of more than a few hundred miles.

2. **Traverse sailing** combines the plane sailing solutions when there are two or more courses and determines the equivalent course and distance made good by a vessel steaming along a series of rhumb lines.
3. **Parallel sailing** is the interconversion of departure and difference of longitude when a vessel is proceeding due east or due west.
4. **Middle- (or mid-) latitude sailing** uses the mean latitude for converting departure to difference of longitude when the course is not due east or due west.
5. **Mercator sailing** provides a mathematical solution of the plot as made on a Mercator chart. It is similar to plane sailing, but uses meridional difference and difference of longitude in place of difference of latitude and departure.
6. **Great circle sailing** involves the solution of courses, distances, and points along a great circle between two points.
7. **Composite sailing** is a modification of great-circle sailing to limit the maximum latitude, generally to avoid ice or severe weather near the poles.

#### 2403. Terms And Definitions

In solutions of the sailings, the following quantities are

used:

1. **Latitude (L)**. The latitude of the point of departure is designated  $L_1$ ; that of the destination,  $L_2$ ; middle (mid) or mean latitude,  $L_m$ ; latitude of the vertex of a great circle,  $L_v$ ; and latitude of any point on a great circle,  $L_x$ .
2. **Mean latitude ( $L_m$ )**. Half the arithmetical sum of the latitudes of two places on the same side of the equator.
3. **Middle or mid latitude ( $L_m$ )**. The latitude at which the arc length of the parallel separating the meridians passing through two specific points is exactly equal to the departure in proceeding from one point to the other. The mean latitude is used when there is no practicable means of determining the middle latitude.
4. **Difference of latitude (l or DLat.)**.
5. **Meridional parts (M)**. The meridional parts of the point of departure are designated  $M_1$ , and of the point of arrival or the destination,  $M_2$ .
6. **Meridional difference (m)**.
7. **Longitude ( $\lambda$ )**. The longitude of the point of departure is designated  $\lambda_1$ ; that of the point of arrival or the destination,  $\lambda_2$ ; of the vertex of a great circle,  $\lambda_v$ ; and of any point on a great circle,  $\lambda_x$ .
8. **Difference of longitude (DLo)**.
9. **Departure (p or Dep.)**.
10. **Course or course angle (Cn or C)**.
11. **Distance (D or Dist.)**.

### GREAT CIRCLE SAILING

#### 2404. Great Circle Sailing By Chart

Navigators can most easily solve great-circle sailing problems graphically. DMAHTC publishes several gnomonic projections covering the principal navigable waters of the world. On these **great circle charts**, any straight line is a great circle. The chart, however, is not conformal; therefore, the navigator cannot directly measure directions and distances as on a Mercator chart.

The usual method of using a gnomonic chart is to plot the route and pick points along the track every  $5^\circ$  of longitude using the latitude and longitude scales in the immediate vicinity of each point. These points are then transferred to a Mercator chart and connected by rhumb lines. The course and distance for each leg is measured on the Mercator chart. See Chapter 25 for a discussion of this process.

#### 2405. Great Circle Sailing By Sight Reduction Tables

Any method of solving a celestial spherical triangle can be used for solving great circle sailing problems. The point of de-

parture replaces the assumed position of the observer, the destination replaces the geographical position of the body, difference of longitude replaces meridian angle or local hour angle, initial course angle replaces azimuth angle, and great circle distance replaces zenith distance ( $90^\circ$  - altitude). See Figure 2405. Therefore, any table of azimuths (if the entering values are meridian angle, declination, and latitude) can be used for determining initial great-circle course. Tables which solve for altitude, such as *Pub. No. 229*, can be used for determining great circle distance. The required distance is  $90^\circ$  - altitude.

In inspection tables such as *Pub. No. 229*, the given combination of  $L_1$ ,  $L_2$ , and DLo may not be tabulated. In this case reverse the name of  $L_2$  and use  $180^\circ$  - DLo for entering the table. The required course angle is then  $180^\circ$  minus the tabulated azimuth, and distance is  $90^\circ$  plus the altitude. If neither combination can be found, solution cannot be made by that method. By interchanging  $L_1$  and  $L_2$ , one can find the supplement of the final course angle.

Solution by table often provides a rapid approximate check, but accurate results usually require triple interpolation. Except for *Pub. No. 229*, inspection tables do not

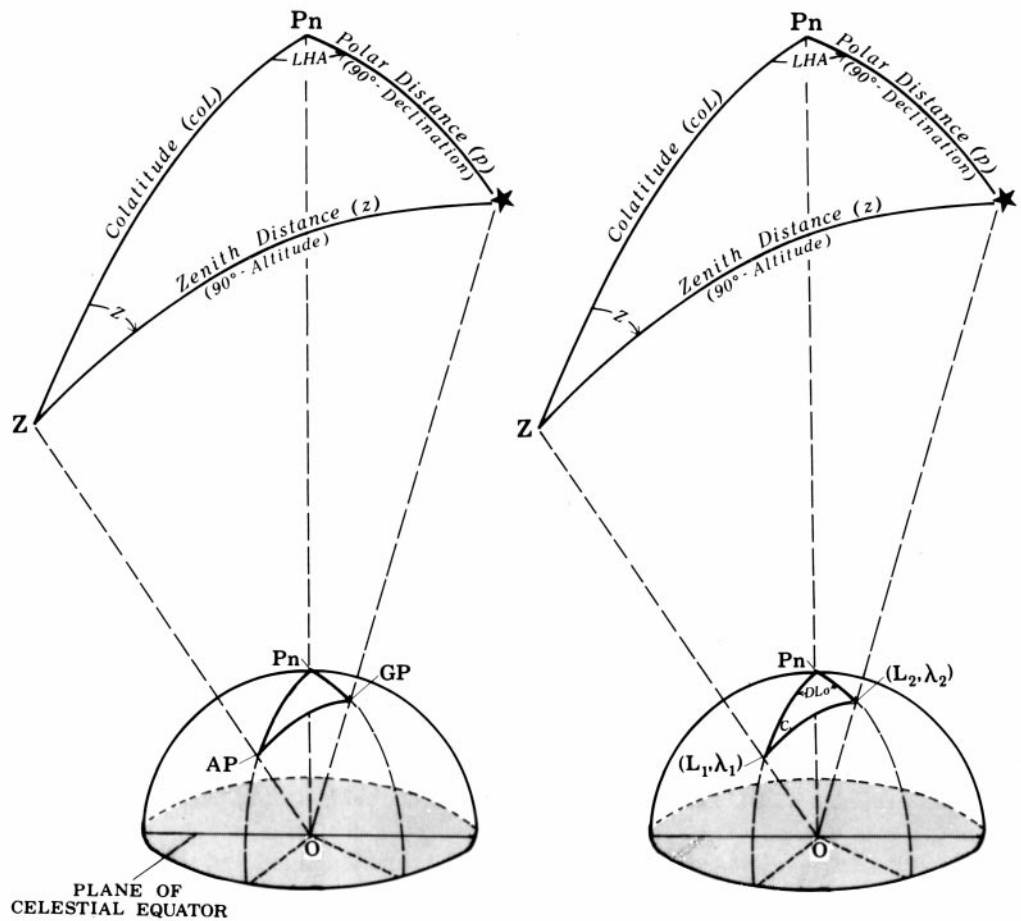


Figure 2405. Adapting the astronomical triangle to the navigational triangle of great circle sailing.

provide a solution for points along the great circle. *Pub. No. 229* provides solutions for these points only if interpolation is not required.

**2406. Great Circle Sailing By *Pub. No. 229***

By entering *Pub. No. 229* with the latitude of the point of departure as latitude, latitude of destination as declination, and difference of longitude as LHA, the tabular altitude and azimuth angle may be extracted and converted to great-circle distance and course. As in sight reduction, the tables are entered according to whether the name of the latitude of the point of departure is the same as or contrary to the name of the latitude of the destination (declination). If the values correspond to those of a celestial body above the celestial horizon,  $90^\circ$  minus the arc of the tabular altitude becomes the distance; the tabular azimuth angle becomes the initial great-circle course angle. If the respondents correspond to those of a celestial body below the celestial horizon, the arc of the tabular altitude plus  $90^\circ$  becomes the distance; the supplement of the tabular azimuth

angle becomes the initial great-circle course angle.

When the Contrary/Same (CS) Line is crossed in either direction, the altitude becomes negative; the body lies below the celestial horizon. For example: If the tables are entered with the LHA (DL<sub>0</sub>) at the bottom of a right-hand page and declination (L<sub>2</sub>) such that the respondents lie above the CS Line, the CS Line has been crossed. Then the distance is  $90^\circ$  plus the tabular altitude; the initial course angle is the supplement of the tabular azimuth angle. Similarly, if the tables are entered with the LHA (DL<sub>0</sub>) at the top of a right-hand page and the respondents are found below the CS Line, the distance is  $90^\circ$  plus the tabular altitude; the initial course angle is the supplement of the tabular azimuth angle. If the tables are entered with the LHA (DL<sub>0</sub>) at the bottom of a right-hand page and the name of L<sub>2</sub> is contrary to L<sub>1</sub>, the respondents are found in the column for L<sub>1</sub> on the facing page. In this case, the CS Line has been crossed; the distance is  $90^\circ$  plus the tabular altitude; the initial course angle is the supplement of the tabular azimuth angle.

The tabular azimuth angle, or its supplement, is prefixed N or S for the latitude of the point of departure and

suffixed E or W depending upon the destination being east or west of the point of departure.

If all entering arguments are integral degrees, the distance and course angle are obtained directly from the tables without interpolation. If the latitude of the destination is nonintegral, interpolation for the additional minutes of latitude is done as in correcting altitude for any declination increment; if the latitude of departure or difference of longitude is nonintegral, the additional interpolation is done graphically.

Since the latitude of destination becomes the declination entry, and all declinations appear on every page, the great circle solution can always be extracted from the volume which covers the latitude of the point of departure.

**Example 1:** Using *Pub. No. 229* find the distance and initial great circle course from lat.  $32^{\circ}\text{S}$ , long.  $116^{\circ}\text{E}$  to lat.  $30^{\circ}\text{S}$ , long.  $31^{\circ}\text{E}$ .

**Solution:** Refer to Figure 2405. The point of departure (lat.  $32^{\circ}\text{S}$ , long.  $116^{\circ}\text{E}$ ) replaces the AP of the observer; the destination (lat.  $30^{\circ}\text{S}$ , long.  $31^{\circ}\text{E}$ ) replaces the GP of the celestial body; the difference of longitude ( $DLo\ 85^{\circ}$ ) replaces local hour angle (LHA) of the body.

Enter *Pub. 229*, Volume 3 with lat.  $32^{\circ}$  (Same Name), LHA  $85^{\circ}$ , and declination  $30^{\circ}$ . The respondents correspond to a celestial body above the celestial horizon. Therefore,  $90^{\circ}$  minus the tabular altitude ( $90^{\circ} - 19^{\circ}12.4' = 70^{\circ}47.6'$ ) becomes the distance; the tabular azimuth angle ( $S66.0^{\circ}\text{W}$ ) becomes the initial great circle course angle, prefixed S for the latitude of the point of departure and suffixed W due to the destination being west of the point of departure.

**Answer:**

$D = 4248$  nautical miles

$C = S66.0^{\circ}\text{W} = 246.0^{\circ}$ .

**Example 2:** Using *Pub. No. 229* find the distance and initial great circle course from lat.  $38^{\circ}\text{N}$ , long.  $122^{\circ}\text{W}$  to lat.  $24^{\circ}\text{S}$ , long.  $151^{\circ}\text{E}$ .

**Solution:** Refer to Figure 2405. The point of departure (lat.  $38^{\circ}\text{N}$ , long.  $122^{\circ}\text{W}$ ) replaces the AP of the observer; the destination (lat.  $24^{\circ}\text{S}$ , long.  $151^{\circ}\text{E}$ ) replaces the GP of the celestial body; the difference of longitude ( $DLo\ 87^{\circ}$ ) replaces local hour angle (LHA) of the body

Enter *Pub. No. 229* Volume 3 with lat.  $38^{\circ}$  (Contrary Name), LHA  $87^{\circ}$ , and declination  $24^{\circ}$ . The respondents correspond to those of a celestial body below the celestial horizon. Therefore, the tabular altitude plus  $90^{\circ}$  ( $12^{\circ}17.0' + 90^{\circ} = 102^{\circ}17.0'$ ) becomes the distance; the supplement of tabular

azimuth angle ( $180^{\circ} - 69.0^{\circ} = 111.0^{\circ}$ ) becomes the initial great circle course angle, prefixed N for the latitude of the point of departure and suffixed W since the destination is west of the point of departure.

Note that the data is extracted from across the CS Line from the entering argument (LHA  $87^{\circ}$ ), indicating that the corresponding celestial body would be below the celestial horizon.

**Answer:**

$D = 6137$  nautical miles

$C = N111.0^{\circ}\text{W} = 249^{\circ}$ .

## 2407. Great Circle Sailing By Computation

In Figure 2407, 1 is the point of departure, 2 the destination, P the pole nearer 1, 1-X-V-2 the great circle through 1 and 2, V the vertex, and X any point on the great circle. The arcs P1, PX, PV, and P2 are the colatitudes of points 1, X, V, and 2, respectively. If 1 and 2 are on opposite sides of the equator, P2 is  $90^{\circ} + L_2$ . The length of arc 1-2 is the great-circle distance between 1 and 2. Arcs 1-2, P1, and P2 form a spherical triangle. The angle at 1 is the initial great-circle course from 1 to 2, that at 2 the supplement of the final great-circle course (or the initial course from 2 to 1), and that at P the DLo between 1 and 2.

Great circle sailing by computation usually involves solving for the initial great circle course; the distance; latitude and longitude, and sometimes the distance, of the vertex; and the latitude and longitude of various points (X) on the great circle. The computation for initial course and the distance involves solution of an oblique spherical triangle, and any method of solv-

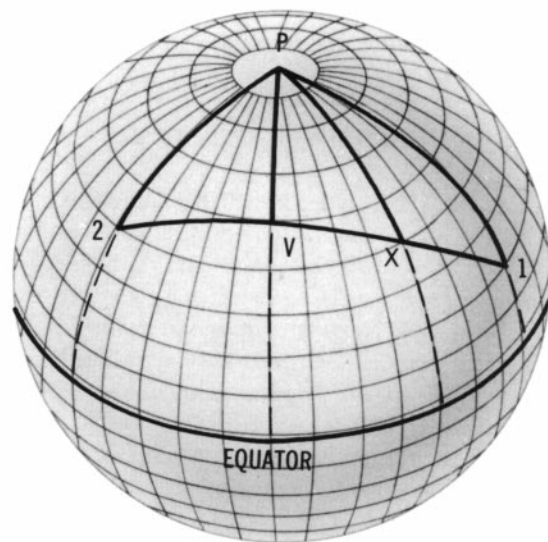


Figure 2407. The navigational triangle and great circle sailing.

ing such a triangle can be used. If 2 is the geographical position (GP) of a celestial body (the point at which the body is in the zenith), this triangle is solved in celestial navigation, except that  $90^\circ - D$  (the altitude) is desired instead of  $D$ . The solution for the vertex and any point  $X$  usually involves the solution of right spherical triangles.

**2408. Points Along The Great Circle**

If the latitude of the point of departure and the initial great-circle course angle are integral degrees, points along the great circle are found by entering the tables with the latitude of departure as the latitude argument (always Same Name), the initial great circle course angle as the LHA argument, and  $90^\circ$  minus distance to a point on the great circle as the declination argument. The latitude of the point on the great circle and the difference of longitude between that point and the point of departure are the tabular altitude and azimuth angle, respectively. If, however, the respondents are extracted from across the CS Line, the tabular altitude corresponds to a latitude on the side of the equator opposite from that of the point of departure; the tabular azimuth angle is the supplement of the difference of longitude.

**Example 1:** Find a number of points along the great circle from latitude  $38^\circ N$ , longitude  $125^\circ W$  when the initial great circle course angle is  $N111^\circ W$ .

**Solution:** Entering the tables with latitude  $38^\circ$  (Same Name), LHA  $111^\circ$ , and with successive declinations of  $85^\circ$ ,  $80^\circ$ ,  $75^\circ$ , etc., the latitudes and differences in longitude from  $125^\circ W$  are found as tabular altitudes and azimuth angles respectively:

**Answer:**

<i>D (NM)</i>	300	600	900	3600
<i>D (arc)</i>	$5^\circ$	$10^\circ$	$15^\circ$	$60^\circ$
<i>dec</i>	$85^\circ$	$80^\circ$	$75^\circ$	$30^\circ$
<i>Lat.</i>	$36.1^\circ N$	$33.9^\circ N$	$31.4^\circ N$	$3.6^\circ N$
<i>Dep.</i>	$125^\circ W$	$125^\circ W$	$125^\circ W$	$125^\circ W$
<i>DLo</i>	$5.8^\circ$	$11.3^\circ$	$16.5^\circ$	$54.1^\circ$
<i>Long</i>	$130.8^\circ W$	$136.3^\circ W$	$141.5^\circ W$	$179.1^\circ$

**Example 2:** Find a number of points along the great circle track from latitude  $38^\circ N$ , long.  $125^\circ W$  when the initial great circle course angle is  $N 69^\circ W$ .

**Solution:** Enter the tables with latitude  $38^\circ$  (Same Name), LHA  $69^\circ$ , and with successive declinations as shown. Find the latitudes and differences of longitude from  $125^\circ W$  as tabular altitudes and azimuth angles, respectively:

**Answer:**

<i>D (NM.)</i>	300	600	900	6600
<i>D (arc)</i>	$5^\circ$	$10^\circ$	$15^\circ$	$110^\circ$
<i>dec</i>	$85^\circ$	$80^\circ$	$75^\circ$	$20^\circ$
<i>Lat.</i>	$39.6^\circ N$	$40.9^\circ N$	$41.9^\circ N$	$3.1^\circ N$
<i>Dep.</i>	$125^\circ W$	$125^\circ W$	$125^\circ W$	$125^\circ W$
<i>DLo</i>	$6.1^\circ$	$12.4^\circ$	$18.9^\circ$	$118.5^\circ$
<i>Long</i>	$131.1^\circ W$	$137.4^\circ W$	$143.9^\circ W$	$116.5^\circ E$

**2409. Finding The Vertex**

Using *Pub. No. 229* to find the approximate position of the vertex of a great circle track provides a rapid check on the solution by computation. This approximate solution is also useful for voyage planning purposes.

Using the procedures for finding points along the great circle, inspect the column of data for the latitude of the point of departure and find the maximum value of tabular altitude. This maximum tabular altitude and the tabular azimuth angle correspond to the latitude of the vertex and the difference of longitude of the vertex and the point of departure.

**Example 1:** Find the vertex of the great circle track from lat.  $38^\circ N$ , long.  $125^\circ W$  when the initial great circle course angle is  $N69^\circ W$ .

**Solution:** Enter *Pub. No. 229* with lat.  $38^\circ$  (Same Name), LHA  $69^\circ$ , and inspect the column for lat.  $38^\circ$  to find the maximum tabular altitude. The maximum altitude is  $42^\circ 38.1'$  at a distance of 1500 nautical miles ( $90^\circ - 65^\circ = 25^\circ$ ) from the point of departure. The corresponding tabular azimuth angle is  $32.4^\circ$ . Therefore, the difference of longitude of vertex and point of departure is  $32.4^\circ$ .

**Answer:**

Latitude of vertex =  $42^\circ 38.1' N$ .  
 Longitude of vertex =  $125^\circ + 32.4^\circ = 157.4^\circ W$ .

**2410. Altering A Great Circle Track To Avoid Obstructions**

Land, ice, or severe weather may prevent the use of great circle sailing for some or all of one's route. One of the principal advantages of solution by great circle chart is that the presence of any hazards is immediately apparent. The pilot charts are particularly useful in this regard. Often a relatively short run by rhumb line is sufficient to reach a point from which the great circle track can be followed. Where a choice is possible, the rhumb line selected should conform as nearly as practicable to the direct great circle.

If the great circle route crosses a navigation hazard,

change the track. It may be satisfactory to follow a great circle to the vicinity of the hazard, one or more rhumb lines along the edge of the hazard, and another great circle to the destination. Another possible solution is the use of composite sailing; still another is the use of two great circles, one from the point of departure to a point near the maximum latitude of unobstructed water and the second from this point to the destination.

### 2411. Composite Sailing

When the great circle would carry a vessel to a higher latitude than desired, a modification of great circle sailing called **composite sailing** may be used to good advantage. The composite track consists of a great circle from the point of departure and tangent to the limiting parallel, a course line along the parallel, and a great circle tangent to the limiting parallel and through the destination.

Solution of composite sailing problems is most easily

made with a great circle chart. For this solution, draw lines from the point of departure and the destination, tangent to the limiting parallel. Then measure the coordinates of various selected points along the composite track and transfer them to a Mercator chart, as in great circle sailing. Composite sailing problems can also be solved by computation, using the equation:

$$\cos DLo_{vx} = \tan L_x \cot L_v$$

The point of departure and the destination are used successively as point X. Solve the two great circles at each end of the limiting parallel, and use parallel sailing along the limiting parallel. Since both great circles have vertices at the same parallel, computation for C, D, and  $DLo_{vx}$  can be made by considering them parts of the same great circle with  $L_1$ ,  $L_2$ , and  $L_v$  as given and  $DLo = DLo_{v1} + DLo_{v2}$ . The total distance is the sum of the great circle and parallel distances.

## TRAVERSE TABLES

### 2412. Using Traverse Tables

**Traverse tables** can be used in the solution of any of the sailings except great circle and composite. They consist of the tabulation of the solutions of plane right triangles. Because the solutions are for integral values of the course angle and the distance, interpolation for intermediate values may be required. Through appropriate interchanges of the headings of the columns, solutions for other than plane sailing can be made. For the solution of the plane right triangle, any value N in the distance (Dist.) column is the hypotenuse; the value opposite in the difference of latitude (D. Lat.) column is the product of N and the cosine of the acute angle; and the other number opposite in the departure (Dep.) column is the product of N and the sine of the acute angle. Or, the number in the D. Lat. column is the value of the side adjacent, and the number in the Dep. column is the value of the side opposite the acute angle. Hence, if the acute angle is the course angle, the side adjacent in the D. Lat. column is meridional difference m; the side opposite in the Dep. column is DLo. If the acute angle is the midlatitude of the formula  $p = DLo \cos L_m$ , then DLo is any value N in the Dist. column, and the departure is the value  $N \times \cos L_m$  in the D. Lat. column.

The examples below clarify the use of the traverse tables for plane, traverse, parallel, mid latitude, and Mercator sailings.

### 2413. Plane Sailing

In plane sailing the figure formed by the meridian through the point of departure, the parallel through the point of arrival, and the course line is considered a plane right tri-

angle. This is illustrated in Figure 2413a.  $P_1$  and  $P_2$  are the points of departure and arrival, respectively. The course angle and the three sides are as labeled. From this triangle:

$$\cos C = \frac{l}{D} \quad \sin C = \frac{p}{D} \quad \tan C = \frac{p}{l}$$

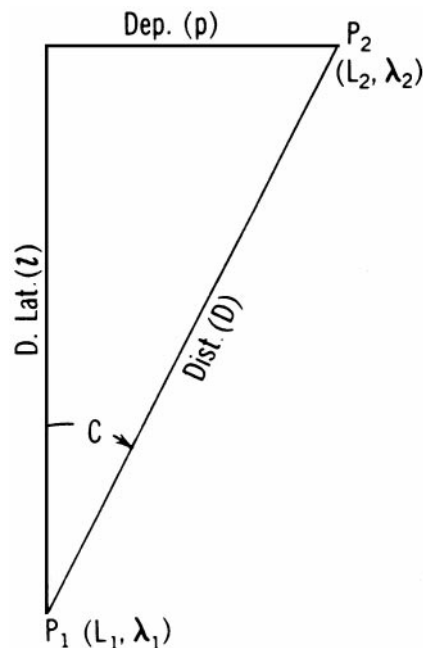


Figure 2413a. The plane sailing triangle.

From the first two of these formulas the following relationships can be derived:

$$l = D \cos C \quad D = l \sec C \quad p = D \sin C.$$

Label *l* as N or S, and *p* as E or W, to aid in identification of the quadrant of the course. Solutions by calculations and traverse tables are illustrated in the following examples:

**Example 1:** A vessel steams 188.0 miles on course 005°.

**Required:** (1) (a) Difference of latitude and (b) departure by computation. (2) (a) difference of latitude and (b) departure by traverse table.

**Solution:**

(1) (a) Difference of latitude by computation:

$$\begin{aligned} \text{diff latitude} &= D \times \cos C \\ &= 188.0 \text{ miles} \times \cos (005^\circ) \\ &= 187.3 \text{ arc min} \\ &= 3^\circ 07.3' N \end{aligned}$$

(1) (b) Departure by computation:

$$\begin{aligned} \text{departure} &= D \times \sin C \\ &= 188.0 \text{ miles} \times \sin (005^\circ) \\ &= 16.4 \text{ miles} \end{aligned}$$

**Answer:**

$$\begin{aligned} \text{Diff. Lat.} &= 3^\circ 07.3' N \\ \text{departure} &= 16.4 \text{ miles} \end{aligned}$$

(2) Difference of latitude and departure by traverse table:

Refer to Figure 2413b. Enter the traverse table and find course 005° at the top of the page. Using the column headings at the top of the table, opposite 188 in the Dist. column extract D. Lat. 187.3 and Dep. 16.4.

(a) D. Lat. = 187.3' N.  
(b) Dep. = 16.4 mi. E.

**Example 2:** A ship has steamed 136.0 miles north and 203.0 miles west.

**Required:** (1) (a) Course and (b) distance by computation. (2) (a) course and (b) distance by traverse table.

**Solution:**

(1) (a) Course by computation:

$$C = \arctan \frac{\text{departure}}{\text{diff. lat.}}$$

TABLE 4												355°   005°									
Traverse 5° Table												185°   175°									
Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.							
1	1.0	0.1	61	60.8	5.3	121	120.5	10.5	181	180.3	15.8	241	240.1	21.0							
2	2.0	0.2	62	61.8	5.4	22	121.5	10.6	82	181.3	15.9	42	241.1	21.1							
3	3.0	0.3	63	62.8	5.5	23	122.5	10.7	83	182.3	15.9	43	242.1	21.2							
4	4.0	0.3	64	63.8	5.6	24	123.5	10.8	84	183.3	16.0	44	243.1	21.3							
5	5.0	0.4	65	64.8	5.7	25	124.5	10.9	85	184.3	16.1	45	244.1	21.4							
6	6.0	0.5	66	65.7	5.8	26	125.5	11.0	86	185.3	16.2	46	245.1	21.4							
7	7.0	0.6	67	66.7	5.8	27	126.5	11.1	87	186.3	16.3	47	246.1	21.5							
8	8.0	0.7	68	67.7	5.9	28	127.5	11.2	88	187.3	16.4	48	247.1	21.6							
9	9.0	0.8	69	68.7	6.0	29	128.5	11.2	89	188.3	16.5	49	248.1	21.7							
10	10.0	0.9	70	69.7	6.1	30	129.5	11.3	90	189.3	16.6	50	249.0	21.8							
60	59.8	5.2	20	119.5	10.5	80	179.3	15.7	40	239.1	20.9	300	298.9	26.1							
Dist.	Dep.	D. Lat.	Dist.	Dep.	D. Lat.	Dist.	Dep.	D. Lat.	Dist.	Dep.	D. Lat.	Dist.	Dep.	D. Lat.							
275°   085°		265°   095°		85°						<table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td>Dist.</td><td>D. Lat.</td><td>Dep.</td></tr> <tr><td>N.</td><td>N x Cos.</td><td>N x Sin.</td></tr> <tr><td>Hypotenuse</td><td>Side Adj.</td><td>Side Opp.</td></tr> </table>			Dist.	D. Lat.	Dep.	N.	N x Cos.	N x Sin.	Hypotenuse	Side Adj.	Side Opp.
Dist.	D. Lat.	Dep.																			
N.	N x Cos.	N x Sin.																			
Hypotenuse	Side Adj.	Side Opp.																			

Figure 2413b. Extract from Table 4.

$$C = \arctan \frac{203.0}{136.0}$$

$$C = N 56^\circ 10.8' W$$

$$C = 304^\circ (\text{to nearest degree})$$

Draw the course vectors to determine the correct course. In this case the vessel has gone north 136 miles and west 203 miles. The course, therefore, must have been between  $270^\circ$  and  $360^\circ$ . No solution other than  $304^\circ$  is reasonable.

(1) (b) Distance by computation:

$$\begin{aligned} D &= \text{diff. latitude} \times \sec C \\ &= 136 \text{ miles} \times \sec (304^\circ) \\ &= 136 \text{ miles} \times 1.8 \\ &= 244.8 \text{ miles} \end{aligned}$$

**Answer:**

$$C = 304^\circ$$

$$D = 244.8 \text{ miles}$$

(2) Solution by traverse table:

Refer to Figure 2413c. Enter the table and find 136 and

203 beside each other in the columns labeled D. Lat. and Dep., respectively. This occurs most nearly on the page for course angle  $56^\circ$ . Therefore, the course is  $304^\circ$ . Interpolating for intermediate values, the corresponding number in the Dist. column is 244.3 miles.

**Answer:**

$$(a) C = 304^\circ$$

$$(b) D = 244.3 \text{ mi.}$$

### 2414. Traverse Sailing

A **traverse** is a series of courses or a track consisting of a number of course lines, such as might result from a sailing vessel beating into the wind. **Traverse sailing** is the finding of a single equivalent course and distance.

Though the problem can be solved graphically on the chart, traverse tables provide a mathematical solution. The distance to the north or south and to the east or west on each course is tabulated, the algebraic sum of difference of latitude and departure is found, and converted to course and distance.

**Example:** A ship steams as follows: course  $158^\circ$ , distance 15.5 miles; course  $135^\circ$ , distance 33.7 miles; course  $259^\circ$ , distance 16.1 miles; course  $293^\circ$ , distance 39.0 miles; course  $169^\circ$ , distance 40.4 miles.

$326^\circ$		$034^\circ$		<b>TABLE 4</b>									$326^\circ$		$034^\circ$		
$214^\circ$		$146^\circ$		<b>Traverse <math>34^\circ</math> Table</b>									$214^\circ$		$146^\circ$		
Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.			
1	0.8	0.6	61	50.6	34.1	121	100.3	67.7	181	150.1	101.2	241	199.8	134.8			
2	1.7	1.1	62	51.4	34.7	22	101.1	68.2	82	150.9	101.8	42	200.6	135.3			
3	2.5	1.7	63	52.2	35.2	23	102.0	68.8	83	151.7	102.3	43	201.5	135.9			
4	3.3	2.2	64	53.1	35.8	24	102.8	69.3	84	152.5	102.9	44	202.3	136.4			
5	4.1	2.8	65	53.9	36.3	25	103.6	69.9	85	153.4	103.5	45	203.1	137.0			
60	49.7	33.6	20	99.5	67.1	80	149.2	100.7	40	199.0	134.2	300	248.7	167.8			
Dist.	Dep.	D. Lat.	Dist.	Dep.	D. Lat.	Dist.	Dep.	D. Lat.	Dist.	Dep.	D. Lat.	Dist.	Dep.	D. Lat.			
$304^\circ$		$056^\circ$		<b><math>56^\circ</math></b>									$236^\circ$		$124^\circ$		
Dist.		D. Lat.		Dep.		N.		N x Cos.		N x Sin.		Hypotenuse		Side Adj.		Side Opp.	

Figure 2413c. Extract from Table 4 .



**Required:** Equivalent single (1) course (2) distance.

**Solution:** Solve each leg as a plane sailing and tabulate each solution as follows. For course 158°, extract the values for D. Lat. and Dep. opposite 155 in the Dist. column. Then, divide the values by 10 and round them off to the nearest tenth. Repeat the procedure for each leg of the vessel's journey.

Course	Dist.	N	S	E	W
degrees	mi.	mi.	mi.	mi.	mi.
158	15.5		14.4	5.8	
135	33.7		23.8	23.8	
259	16.1		3.1		15.8
293	39.0	15.2			35.9
169	40.4		39.7	7.7	
Subtotals		15.2	81.0	37.3	51.7
			-15.2		-37.3
N/S Total			65.8 S		14.4 W

Thus, the latitude difference is S 65.8 miles and the departure is W 14.4 miles. Convert this to a course and distance using the formulas discussed in section 2413.

**Answer:**

- (1) C = 192.3°
- (2) D = 67.3 miles.

**2415. Parallel Sailing**

Parallel sailing consists of the interconversion of departure and difference of longitude. It is the simplest form of spherical sailing. The formulas for these transformations are:

$$DLo = p \sec L \quad p = DLo \cos L$$

**Example 1:** The DR latitude of a ship on course 090° is 49°30' N. The ship steams on this course until the longitude changes 3°30'.

**Required:** The departure by (1) computation and (2) traverse table.

**Solution:**

(1) Solution by computation:

$$DLo = 3^\circ 30'$$

$$\begin{aligned} DLo &= 210 \text{ arc min} \\ p &= DLo \times \cos L \\ p &= 210 \text{ arc minutes} \times \cos (49.5^\circ) \\ p &= 136.4 \text{ miles} \end{aligned}$$

**Answer:**

$$p = 136.4 \text{ miles}$$

(2) Solution by traverse table:

Refer to Figure 2415a. Enter the traverse table with latitude as course angle and substitute DLo as the heading of the Dist. column and Dep. as the heading of the D. Lat. column. Since the table is computed for integral degrees of course angle (or latitude), the tabulations in the pages for 49° and 50° must be interpolated for the intermediate value (49°30'). The departure for latitude 49° and DLo 210' is 137.8 miles. The departure for latitude 50° and DLo 210' is 135.0 miles. Interpolating for the intermediate latitude, the departure is 136.4 miles.

**Answer:**

$$p = 136.4 \text{ miles}$$

**Example 2:** The DR latitude of a ship on course 270° is 38°15'S. The ship steams on this course for a distance of 215.5 miles.

**Required:** The change in longitude by (1) computation and (2) traverse table.

**Solution:**

(1) Solution by computation

$$\begin{aligned} DLo &= 215.5 \text{ arc min} \times \sec (38.25^\circ) \\ DLo &= 215.5 \text{ arc min} \times 1.27 \\ DLo &= 274.4 \text{ minutes of arc (west)} \\ DLo &= 4^\circ 34.4' W \end{aligned}$$

**Answer:**

$$DLo = 4^\circ 34.4' W$$

(2) Solution by traverse table

Refer to Figure 2415b. Enter the traverse tables with latitude as course angle and substitute DLo as the heading of the Dist. column and Dep. as the heading of the D. Lat. column. As the table is computed for integral degrees of course angle (or latitude), the tabulations in the pages for 38° and 39° must be interpolated for the minutes of latitude. Corresponding to Dep. 215.5 miles in the former is DLo

$\frac{319^\circ}{221^\circ}$		$\frac{041^\circ}{139^\circ}$		<b>TABLE 4</b>									$\frac{319^\circ}{221^\circ}$		$\frac{041^\circ}{139^\circ}$			
Traverse		<b>41°</b>		Table														
Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.				
21	15.8	13.8	81	61.1	53.1	141	106.4	92.5	201	151.7	131.9	261	197.0	171.2				
22	16.6	14.4	82	61.9	53.8	42	107.2	93.2	02	152.5	132.5	62	197.7	171.9				
23	17.4	15.1	83	62.6	54.5	43	107.9	93.8	03	153.2	133.2	63	198.5	172.5				
24	18.1	15.7	84	63.4	55.1	44	108.7	94.5	04	154.0	133.8	64	199.2	173.2				
25	18.9	16.4	85	64.2	55.8	45	109.4	95.1	05	154.7	134.5	65	200.0	173.9				
26	19.6	17.1	86	64.9	56.4	46	110.2	95.8	06	155.5	135.1	66	200.8	174.5				
27	20.4	17.7	87	65.7	57.1	47	110.9	96.4	07	156.2	135.8	67	201.5	175.2				
28	21.1	18.4	88	66.4	57.7	48	111.7	97.1	08	157.0	136.5	68	202.3	175.8				
29	21.9	19.0	89	67.2	58.4	49	112.5	97.8	09	157.7	137.1	69	203.0	176.5				
30	22.6	19.7	90	67.9	59.0	50	113.2	98.4	10	158.5	137.8	70	203.8	177.1				
Dist.	Dep.	D. Lat.	Dist.	Dep.	D. Lat.	Dist.	Dep.	D. Lat.	Dist.	Dep.	D. Lat.	Dist.	Dep.	D. Lat.				
$\frac{311^\circ}{229^\circ}$		$\frac{049^\circ}{131^\circ}$		<b>49°</b>									Dist.		D. Lat.		Dep.	
													D Lo		Dep.			
													m		D Lo			

Figure 2415a. Extract from Table 4.

$\frac{322^\circ}{218^\circ}$		$\frac{038^\circ}{142^\circ}$		<b>TABLE 4</b>									$\frac{322^\circ}{218^\circ}$		$\frac{038^\circ}{142^\circ}$			
Traverse		<b>38°</b>		Table														
Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.				
81	24.4	19.1	91	71.7	56.0	151	119.0	93.0	211	166.3	129.9	271	213.6	168.8				
82	25.2	19.7	92	72.5	56.6	52	119.8	93.6	12	167.1	130.5	72	214.3	167.5				
83	26.0	20.3	93	73.3	57.3	53	120.6	94.2	13	167.8	131.1	73	215.1	168.1				
84	26.8	20.9	94	74.1	57.9	54	121.4	94.8	14	168.6	131.8	74	215.9	168.7				
85	27.6	21.5	95	74.9	58.5	55	122.1	95.4	15	169.4	132.4	75	216.7	169.3				
Dist.	Dep.	D. Lat.	Dist.	Dep.	D. Lat.	Dist.	Dep.	D. Lat.	Dist.	Dep.	D. Lat.	Dist.	Dep.	D. Lat.				
													D Lo		Dep.			
													m		D Lo			

Figure 2415b. Extract from Table 4.

273.5', and in the latter DLo 277.3'. Interpolating for minutes of latitude, the DLo is 274.4'W.

**Answer:**

$$DLo = 4^\circ 34.4'$$

**2416. Middle-Latitude Sailing**

Middle-latitude sailing combines plane sailing and parallel sailing. Plane sailing is used to find difference of latitude and departure when course and distance are known, or vice versa. Parallel sailing is used to interconvert departure and difference of longitude. The mean latitude ( $L_m$ ) is normally used for want of a practicable means of determining the middle latitude, the

latitude at which the arc length of the parallel separating the meridians passing through two specific points is exactly equal to the departure in proceeding from one point to the other. The formulas for these transformations are:

$$DLo = p \sec L_m \quad p = DLo \cos L_m.$$

The mean latitude ( $L_m$ ) is half the arithmetical sum of the latitudes of two places on the same side of the equator. It is labeled N or S to indicate its position north or south of the equator. If a course line crosses the equator, solve each course line segment separately.

**Example 1:** A vessel steams 1,253 miles on course 070°

from lat.  $15^{\circ}17.0' N$ , long.  $151^{\circ}37.0' E$ .

**Required:** Latitude and longitude of the point of arrival by (1) computation and (2) traverse table.

**Solution:**

(1) Solution by computation:

$$l = D \cos C; p = D \sin C; \text{ and } DLo = p \sec L_m$$

$$D = 1253.0 \text{ miles.}$$

$$C = 070^{\circ}$$

$$l = 428.6' N$$

$$p = 1177.4 \text{ miles } E$$

$$L_1 = 15^{\circ}17.0' N$$

$$l = 7^{\circ}08.6' N$$

$$L_2 = 22^{\circ}25.6' N$$

$$L_m = 18^{\circ}51.3' N$$

$$DLo = 1244.2' E$$

$$\lambda_1 = 151^{\circ}37.0' E$$

$$DLo = 20^{\circ}44.2' E$$

$$\lambda_2 = 172^{\circ}21.2' E$$

**Answer:**

$$L_2 = 22^{\circ}25.6' N$$

$$\lambda_2 = 172^{\circ}21.2' E$$

(2) Solution by traverse tables:

Refer to Figure 2416a. Enter the traverse table with course  $070^{\circ}$  and distance 1,253 miles. Because a number as high as 1,253 is not tabulated in the Dist. column, obtain the values for D. Lat. and Dep. for a distance of 125.3 miles and multiply them by 10. Interpolating between the tabular distance arguments yields D. Lat. = 429' and Dep. = 1,178 miles. Converting the D. Lat. value to degrees of latitude yields  $7^{\circ}09.0'$ . The point of arrival's latitude, therefore, is  $22^{\circ}26' N$ . This results in a mean latitude of  $18^{\circ}51.5' N$ .

Reenter the table with the mean latitude as course angle and substitute DLo as the heading of the Dist. column and Dep. as the heading of the D. Lat. column. Since the table is computed for integral degrees of course angle (or latitude), the tabulations in the pages for  $18^{\circ}$  and  $19^{\circ}$  must be

interpolated for the minutes of  $L_m$ . In the  $18^{\circ}$  table, interpolate for DLo between the departure values of 117.0 miles and 117.9 miles. This results in a DLo value of 123.9. In the  $19^{\circ}$  table, interpolate for DLo between the departure values of 117.2 and 118.2. This yields a DLo value of 124.6.

Having obtained the DLo values corresponding to mean latitudes of  $18^{\circ}$  and  $19^{\circ}$ , interpolate for the actual value of the mean latitude:  $18^{\circ}51.5' N$ . This yields the value of DLo: 124.5. Multiply this final value by ten to obtain  $DLo = 1245 \text{ minutes} = 20^{\circ}45' E$ .

Add the changes in latitude and longitude to the original position's latitude and longitude to obtain the final position.

**Answer:**

$$L_2 = 22^{\circ}26' N$$

$$\lambda_2 = 172^{\circ}22.0' E$$

**Example 2:** A vessel at lat.  $8^{\circ}48.9' S$ , long.  $89^{\circ}53.3' W$  is to proceed to lat.  $17^{\circ}06.9' S$ , long.  $104^{\circ}51.6' W$ .

**Required:** Course and distance by (1) computation and (2) traverse table.

**Solution:**

(1) Solution by computation:

$$p = DLo \cos L_m; \tan C = \frac{p}{l}; \text{ and } D = l \sec C$$

$$DLo = 14^{\circ}58.3'$$

$$DLo = 898.3'$$

$$L_m = 12^{\circ}57.9' S$$

$$p = 893.8 \text{ arc min} \times \cos (12^{\circ}57.9')$$

$$p = 875.4 \text{ arc min}$$

$$l = 17.1^{\circ} - 8.8^{\circ}$$

$$l = 8.3^{\circ}$$

$$l = 498 \text{ arc min}$$

$$C = \arctan \frac{875.4 \text{ arc min}}{498 \text{ arc min}}$$

$$C = S 60.4^{\circ} W$$

$$C = 240.4^{\circ}$$

$$D = 498 \text{ arc min} \times \sec (60.4^{\circ})$$

$$D = 1008.2 \text{ miles}$$

<b>340°</b>		<b>020°</b>		<b>TABLE 4</b>									<b>340°</b>		<b>020°</b>	
<b>200°</b>		<b>160°</b>		<b>Traverse 20° Table</b>									<b>200°</b>		<b>160°</b>	
Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.		
<b>1</b>	0.9	0.3	<b>61</b>	57.3	20.9	<b>121</b>	113.7	41.4	<b>181</b>	170.1	61.9	<b>241</b>	226.5	82.4		
<b>2</b>	1.9	0.7	<b>62</b>	58.3	21.2	<b>22</b>	114.6	41.7	<b>82</b>	171.0	62.2	<b>42</b>	227.4	82.8		
<b>3</b>	2.8	1.0	<b>63</b>	59.2	21.5	<b>23</b>	115.6	42.1	<b>83</b>	172.0	62.6	<b>43</b>	228.3	83.1		
<b>4</b>	3.8	1.4	<b>64</b>	60.1	21.9	<b>24</b>	116.5	42.4	<b>84</b>	172.9	62.9	<b>44</b>	229.3	83.5		
<b>5</b>	4.7	1.7	<b>65</b>	61.1	22.2	<b>25</b>	117.5	42.8	<b>85</b>	173.8	63.3	<b>45</b>	230.2	83.8		
<b>6</b>	5.6	2.1	<b>66</b>	62.0	22.6	<b>26</b>	118.4	43.1	<b>86</b>	174.8	63.6	<b>46</b>	231.2	84.1		
<b>7</b>	6.6	2.4	<b>67</b>	63.0	22.9	<b>27</b>	119.3	43.4	<b>87</b>	175.7	64.0	<b>47</b>	232.1	84.5		

Dist.	Dep.	D. Lat.	Dist.	Dep.	D. Lat.	Dist.	Dep.	D. Lat.	Dist.	Dep.	D. Lat.	Dist.	Dep.	D. Lat.		
<b>290°</b>		<b>070°</b>		<b>70°</b>									<b>290°</b>		<b>070°</b>	
<b>250°</b>		<b>110°</b>				Dist.		D. Lat.		Dep.						
						N.		N x Cos.		N x Sin.						
						Hypotenuse		Side Adj.		Side Opp.						

<b>342°</b>		<b>018°</b>		<b>TABLE 4</b>									<b>342°</b>		<b>018°</b>	
<b>198°</b>		<b>162°</b>		<b>Traverse 18° Table</b>									<b>198°</b>		<b>162°</b>	
Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.		
<b>1</b>	1.0	0.3	<b>61</b>	58.0	18.9	<b>121</b>	115.1	37.4	<b>181</b>	172.1	55.9	<b>241</b>	229.2	74.5		
<b>2</b>	1.9	0.6	<b>62</b>	59.0	19.2	<b>22</b>	116.0	37.7	<b>82</b>	173.1	56.2	<b>42</b>	230.2	74.8		
<b>3</b>	2.9	0.9	<b>63</b>	59.9	19.5	<b>23</b>	117.0	38.0	<b>83</b>	174.0	56.6	<b>43</b>	231.1	75.1		
<b>4</b>	3.8	1.2	<b>64</b>	60.9	19.8	<b>24</b>	117.9	38.3	<b>84</b>	175.0	56.9	<b>44</b>	232.1	75.4		
<b>5</b>	4.8	1.5	<b>65</b>	61.8	20.1	<b>25</b>	118.9	38.6	<b>85</b>	175.9	57.2	<b>45</b>	233.0	75.7		

Dist.	Dep.	D. Lat.	Dist.	Dep.	D. Lat.	Dist.	Dep.	D. Lat.	Dist.	Dep.	D. Lat.	Dist.	Dep.	D. Lat.		
<b>288°</b>		<b>072°</b>		<b>72°</b>									<b>288°</b>		<b>072°</b>	
<b>252°</b>		<b>108°</b>				Dist.		D. Lat.		Dep.						
						D Lo		Dep.								
						m		D Lo								

<b>341°</b>		<b>019°</b>		<b>TABLE 4</b>									<b>341°</b>		<b>019°</b>	
<b>199°</b>		<b>161°</b>		<b>Traverse 19° Table</b>									<b>199°</b>		<b>161°</b>	
Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.		
<b>1</b>	0.9	0.3	<b>61</b>	57.7	19.9	<b>121</b>	114.4	39.4	<b>181</b>	171.1	58.9	<b>241</b>	227.9	78.5		
<b>2</b>	1.9	0.7	<b>62</b>	58.6	20.2	<b>22</b>	115.4	39.7	<b>82</b>	172.1	59.3	<b>42</b>	228.8	78.8		
<b>3</b>	2.8	1.0	<b>63</b>	59.6	20.5	<b>23</b>	116.3	40.0	<b>83</b>	173.0	59.6	<b>43</b>	229.8	79.1		
<b>4</b>	3.8	1.3	<b>64</b>	60.5	20.8	<b>24</b>	117.2	40.4	<b>84</b>	174.0	59.9	<b>44</b>	230.7	79.4		
<b>5</b>	4.7	1.6	<b>65</b>	61.5	21.2	<b>25</b>	118.2	40.7	<b>85</b>	174.9	60.2	<b>45</b>	231.7	79.8		

Dist.	Dep.	D. Lat.	Dist.	Dep.	D. Lat.	Dist.	Dep.	D. Lat.	Dist.	Dep.	D. Lat.	Dist.	Dep.	D. Lat.		
<b>289°</b>		<b>071°</b>		<b>71°</b>									<b>289°</b>		<b>071°</b>	
<b>251°</b>		<b>109°</b>				Dist.		D. Lat.		Dep.						
						D Lo		Dep.								
						m		D Lo								

Figure 2416a. Extracts from the Table 4.

**Answer:**

$C = 240.4^\circ$   
 $D = 1008.2 \text{ miles}$

The labels (N, S, E, W) of  $l$ ,  $p$ , and  $C$  are determined by noting the direction of motion or the relative positions of the two places.

(2) Solution by traverse tables:

Refer to Figure 2416b. Enter the traverse table with the mean latitude as course angle and substitute  $DLo$  as the heading of the Dist. column and Dep. as the heading of the D. Lat. column. Since the table is computed for integral values of course angle (or latitude), it is usually necessary to extract the value of departure for values just less and just greater than the  $L_m$  and then interpolate for the minutes of  $L_m$ . In this case where  $L_m$  is almost  $13^\circ$ , enter the table with  $L_m$   $13^\circ$  and  $DLo$  898.3' to find Dep. 875 miles. The departure is found for  $DLo$  89.9', and then multiplied by 10.

Reenter the table to find the numbers 875 and 498 beside each other in the columns labeled Dep. and D. Lat., respectively. Because these high numbers are not tabulated, divide them by 10, and find 87.5 and 49.8. This occurs most nearly on the page for course angle  $60^\circ$  (fig. 2414c). Interpolating for intermediate values, the corresponding number in the Dist. column is about 100.5. Multiplying this by 10, the distance is about 1005 miles.

**Answer:**

$C = 240^\circ$   
 $D = 1005 \text{ miles.}$

The labels (N, S, E, W) of  $l$ ,  $p$ ,  $DLo$ , and  $C$  are determined by noting the direction of motion or the relative positions of the two places.

**2417. Mercator Sailing**

Mercator sailing problems can be solved graphically on a Mercator chart. For mathematical solution, the formulas of Mercator sailing are:

$$\tan C = \frac{DLo}{m} \quad DLo = m \tan C.$$

After solving for course angle by Mercator sailing, solve for distance using the plane sailing formula:

$$D = l \sec C$$

**Example 1:** A ship at lat.  $32^\circ 14.7'N$ , long.  $66^\circ 28.9'W$  is to head for a point near Chesapeake Light, lat.  $36^\circ 58.7'N$ , long.  $75^\circ 42.2'W$ .

**Required:** Course and distance by (1) computation and (2) traverse table.

**Solution:**

(1) Solution by computation:

$$\tan C = \frac{DLo}{m}, \text{ and } D = l \sec C.$$

First calculate the meridional difference by entering Table 6 and interpolating for the meridional parts for the original and final latitudes. The meridional difference is the difference between these two val-

TABLE 4													347°   013°		
193°   167°			Traverse 13° Table							193°   167°					
Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.	
27	26.3	6.1	87	84.8	19.6	47	143.2	33.1	07	201.7	46.6	67	260.2	60.1	
28	27.3	6.3	88	85.7	19.8	48	144.2	33.3	08	202.7	46.8	68	261.1	60.3	
29	28.3	6.5	89	86.7	20.0	49	145.2	33.5	09	203.6	47.0	69	262.1	60.5	
30	29.2	6.7	90	87.7	20.2	50	146.2	33.7	10	204.6	47.2	70	263.1	60.7	

Dist.	Dep.	D. Lat.	Dist.	Dep.	D. Lat.	Dist.	Dep.	D. Lat.	Dist.	Dep.	D. Lat.	Dist.	Dep.	D. Lat.	
283°   077°						77°									
257°   103°															

Dist.	D. Lat.	Dep.
D Lo	Dep.	
	m	D Lo

Figure 2416b. Extract from Table 4.

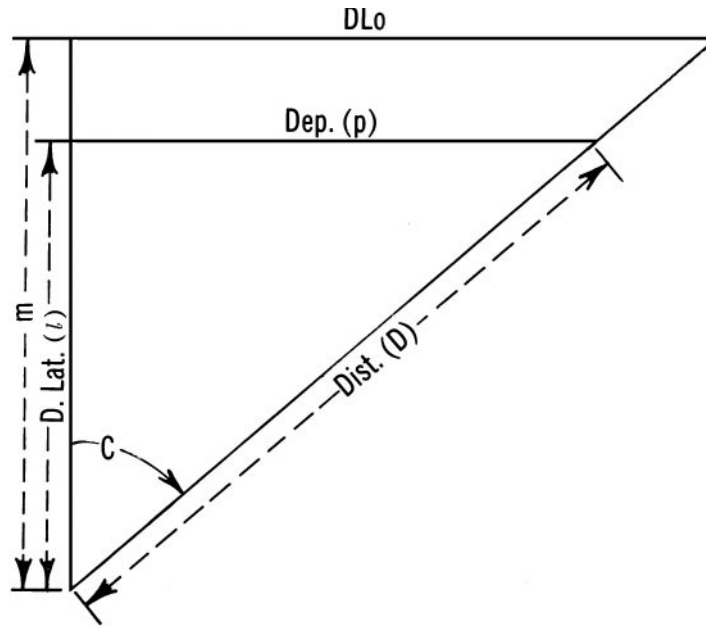


Figure 2417a. Mercator and plane sailing relationships.

ues. Having calculated the meridional difference, simply solve for course and distance from the equations above.

$$\begin{aligned}
 M_2 (36^\circ 58.7' N) &= 2377.5 \\
 M_1 (32^\circ 14.7' N) &= 2032.9 \\
 m &= 344.6 \\
 \\ 
 \lambda_2 &= 075^\circ 42.2' W \\
 \lambda_1 &= 066^\circ 28.9' W \\
 DLo &= 9^\circ 13.3' W \\
 DLo &= 553.3' W \\
 \\ 
 C &= \arctan (553.3 \div 344.6) \\
 C &= N 58.1^\circ W \\
 C &= 301.9^\circ \\
 \\ 
 L_2 &= 36^\circ 58.7' N \\
 L_1 &= 32^\circ 14.7' N \\
 l &= 4^\circ 44.0' N \\
 l &= 284.0' \\
 \\ 
 D &= 284.0 \text{ arc min} \times \sec (58.1^\circ) \\
 D &= 537.4 \text{ miles}
 \end{aligned}$$

**Answer:**

$$C = 301.9^\circ$$

$$D = 537.4 \text{ miles}$$

(2) Solution by traverse table:

Refer to Figure 2417b. Substitute  $m$  as the heading of the D. Lat. column and  $DLo$  as the heading of the Dep. column. Inspect the table for the numbers 343.7 and 553.3 in the columns relabeled  $m$  and  $DLo$ , respectively.

Because a number as high as 343.7 is not tabulated in the  $m$  column, it is necessary to divide  $m$  and  $DLo$  by 10. Then inspect to find 34.4 and 55.3 abreast in the  $m$  and  $DLo$  columns, respectively. This occurs most nearly on the page for course angle  $58^\circ$  or course  $302^\circ$ .

Reenter the table with course  $302^\circ$  to find Dist. for D. Lat.  $284.0'$ . This distance is 536 miles.

**Answer:**

$$C = 302^\circ$$

$$D = 536 \text{ miles}$$

**Example 2:** A ship at lat.  $75^\circ 31.7' N$ , long.  $79^\circ 08.7' W$ , in Baffin Bay, steams 263.5 miles on course  $155^\circ$ .

**Required:** Latitude and longitude of point of arrival by (1) computation and (2) traverse table.

**Solution:**

(1) Solution by computation:

$$l = D \cos C; \text{ and } DLo = m \tan C$$

TABLE 4														
328°   032°			Traverse 32° Table									328°   032°		
212°   148°												212°   148°		
Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.
1	0.8	0.5	61	51.7	32.3	121	102.6	64.1	181	153.5	96.9	241	204.4	127.7
2	1.7	1.1	62	52.6	32.9	22	103.5	64.7	82	154.3	96.4	42	205.2	128.2
3	2.5	1.6	63	53.4	33.4	23	104.3	65.2	83	155.2	97.0	43	206.1	128.8
4	3.4	2.1	64	54.8	33.9	24	105.2	65.7	84	156.0	97.5	44	206.9	129.3
5	4.2	2.6	65	55.1	34.4	25	106.0	66.2	85	156.9	98.0	45	207.8	129.8
6	5.1	3.2	66	56.0	35.0	26	106.9	66.8	86	157.7	98.6	46	208.6	130.4

351	297.7	186.0	411	348.5	217.8	471	399.4	249.6	531	450.3	281.4	591	501.2	313.2
52	298.5	186.5	12	349.4	218.3	72	400.3	250.1	32	451.2	281.9	92	502.0	313.7
53	299.4	187.1	13	350.2	218.9	73	401.1	250.7	33	452.0	282.4	93	502.9	314.2
54	300.2	187.6	14	351.1	219.4	74	402.0	251.2	34	452.9	283.0	94	503.7	314.8
55	301.1	188.1	15	351.9	219.9	75	402.8	251.7	35	453.7	283.5	95	504.6	315.3
56	301.9	188.7	16	352.8	220.4	76	403.7	252.2	36	454.6	284.0	96	505.4	315.8
57	302.8	189.2	17	353.6	221.0	77	404.5	252.8	37	455.4	284.6	97	506.3	316.4
58	303.6	189.7	18	354.5	221.5	78	405.4	253.3	38	456.2	285.1	98	507.1	316.9
59	304.4	190.2	19	355.3	222.0	79	406.2	253.8	39	457.1	285.6	99	508.0	317.4
60	305.3	190.8	20	356.2	222.6	80	407.1	254.4	40	457.9	286.2	600	508.8	318.0

302°   058°			58°			Dist.   D. Lat.   Dep.		
238°   122°						D Lo   Dep.   m   D Lo		

Figure 2417b. Extract from Table 4 composed of parts of left and right hand pages for course angle 58°.

- $D = 263.5 \text{ mi.}$
- $C = 155^\circ$
- $l = 238.8' \text{ S}$
- $l = 3^\circ 58.8' \text{ S}$
  
- $L_1 = 75^\circ 31.7' \text{ N}$
- $l = 3^\circ 58.8' \text{ S}$
- $L_2 = 71^\circ 32.9' \text{ N}$
  
- $M_1 = 7072.4$
- $M_2 = 6226.1$
- $m = 846.3$
  
- $DLo = 394.6' \text{ E}$
- $DLo = 6^\circ 34.6' \text{ E}$
  
- $\lambda_1 = 79^\circ 08.7' \text{ W}$
- $DLo = 6^\circ 34.6' \text{ E}$
- $l_2 = 072^\circ 34.1' \text{ W}$

The labels (N, S, E, W) of  $l$ ,  $DLo$ , and  $C$  are determined by noting the direction of motion or the relative positions of the two places.

**Answer:**

$L_2 = 71^\circ 32.9'$

$\lambda_2 = 072^\circ 34.1'$

(2) Solution by traverse table:

Refer to Figure 2417c . Enter the traverse table with course  $155^\circ$  and Dist. 263.5 miles to find D. Lat. 238.8'. The latitude of the point of arrival is found by subtracting the D. Lat. from the latitude of the point of departure. Determine the meridional difference by Table Table 4 ( $m = 846.3$ ).

Reenter the table with course  $155^\circ$  to find the  $DLo$  corresponding to  $m = 846.3$ . Substitute meridional difference  $m$  as the heading of the D. Lat. column and  $DLo$  as the heading of the Dep. column. Because a number as high as 846.3 is not tabulated in the  $m$  column, divide  $m$  by 10 and then inspect the  $m$  column for a value of 84.6. Interpolating as necessary, the latter value is opposite  $DLo$  39.4'. The  $DLo$  is 394' ( $39.4' \times 10$ ). The longitude of the point of arrival is found by applying the  $DLo$  to the longitude of the point of departure.

**Answer:**

$L_2 = 71^\circ 32.9' \text{ N.}$

$\lambda_2 = 72^\circ 34.7' \text{ W.}$

335°   025°			TABLE 4									335°   025°		
205°   155°			Traverse 25° Table									205°   155°		
Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.	Dist.	D. Lat.	Dep.
21	19.0	8.9	81	73.4	34.2	141	127.8	59.6	201	182.2	84.9	261	236.5	110.8
22	19.9	9.3	82	74.3	34.7	42	128.7	60.0	02	183.1	85.4	62	237.5	110.7
23	20.8	9.7	83	75.2	35.1	43	129.6	60.4	03	184.0	85.8	63	238.4	111.1
24	21.8	10.1	84	76.1	35.5	44	130.5	60.9	04	184.9	86.2	64	239.3	111.6
25	22.7	10.6	85	77.0	35.9	45	131.4	61.3	05	185.8	86.6	65	240.2	112.0
33	29.9	13.9	93	84.3	39.3	53	138.7	64.7	13	193.0	90.0	73	247.4	115.4
34	30.8	14.4	94	85.2	39.7	54	139.6	65.1	14	193.9	90.4	74	248.3	115.8
Dist.	Dep.	D. Lat.	Dist.	Dep.	D. Lat.	Dist.	Dep.	D. Lat.	Dist.	Dep.	D. Lat.	Dist.	Dep.	D. Lat.
295°   065°			65°									245°   115°		
Dist.		D. Lat.	Dep.		D. Lo	Dist.		D. Lat.	Dep.		D. Lo	Dist.		D. Lat.
D. Lo		Dep.	m		D. Lo	D. Lo		D. Lo		D. Lo	D. Lo		D. Lo	

Figure 2417c. Extract from Table 4.

**2418. Additional Problems**

**Example:** A vessel steams 117.3 miles on course 214°.  
**Required:** (1) Difference of latitude, (2) departure, by plane sailing.  
**Answers:** (1) l 97.2'S, (2) p 65.6 mi. W.

**Example:** A steamer is bound for a port 173.3 miles south and 98.6 miles east of the vessel's position  
**Required:** (1) Course, (2) distance, by plane sailing.  
**Answers:** (1) C 150.4°; (2) D 199.4 mi. by computation, 199.3 mi. by traverse table.

**Example:** A ship steams as follows: course 359°, distance 28.8 miles; course 006°, distance 16.4 miles; course 266°, distance 4.9 miles; course 144°, distance 3.1 miles; course 333°, distance 35.8 miles; course 280°, distance 19.3 miles.  
**Required:** (1) Course, (2) distance, by traverse sailing.  
**Answers:** (1) C 334.4°, (2) D 86.1 mi.

**Example:** The 1530 DR position of a ship is lat. 44°36.3'N, long. 31°18.3'W. The ship is on course 270°, speed 17 knots.  
**Required:** The 2000 DR position, by parallel sailing.  
**Answer:** 2000 DR: L 44°36.3'N, λ 33°05.7'W.

**Example:** A ship at lat. 33°53.3'S, long. 18°23.1'E, leaving Cape Town, heads for a destination near Ambrose Light, lat. 40°27.1'N, long. 73°49.4'W.  
**Required:** (1) Course and (2) distance, by Mercator sailing.  
**Answers:** (1) C 310.9°; (2) D 6,811.5 mi. by computation, 6,812.8 mi. by traverse table.

**Example:** A ship at lat. 15°03.7'N, long. 151°26.8'E steams 57.4 miles on course 035°.  
**Required:** (1) Latitude and (2) longitude of the point of arrival, by Mercator sailing.  
**Answers:** (1) L 15°50.7'N; (2) λ 152°00.7'E.