

Human Capital

Modify the Solow model to include *human capital*. This allows the skills of workers to increase, separately from technological progress.

$$Y = K^\alpha (AH)^{1-\alpha} \quad (1)$$

where

$$H = e^{\psi u} L. \quad (2)$$

L is the number of workers. u is the amount of time spent acquiring human capital (think of it as years of schooling).

In per worker terms,

$$h = e^{\psi u}. \quad (3)$$

Return to u

What is ψ ? The increase in H from one more unit of time acquiring human capital. Take total derivative:

$$dH = \psi e^{\psi u} L du = (\psi H) du \quad (4)$$

This is the absolute change in H given an increase in u . The proportional change in H is

$$\frac{dH}{H} = \psi du. \quad (5)$$

If $\psi = 0.10$, then this says that if $du = 1$, then H rises by 10%. This formula for human capital is consistent with micro-level evidence on wages and earnings. ψ is the return to one year of schooling.

The Modified Solow Model

Start by writing output in per worker terms

$$y = k^\alpha (Ah)^{1-\alpha}. \quad (6)$$

Take logs and derivatives,

$$\frac{\dot{y}}{y} = \alpha \frac{\dot{k}}{k} + (1 - \alpha) \frac{\dot{A}}{A} + (1 - \alpha) \frac{\dot{h}}{h}. \quad (7)$$

We assume that

$$\frac{\dot{h}}{h} = 0 \quad (8)$$

$$\frac{\dot{A}}{A} = g \quad (9)$$

or human capital (h) does not have trend growth, but there is trend growth in technology (A).

A balanced growth path, as before, is where \dot{y}/y is constant. That required that $\dot{y}/y = \dot{k}/k$. So again we have that

$$\frac{\dot{y}}{y} = g \quad (10)$$

along the balanced growth path. Human capital doesn't change this.

Steady state

As before, along the balanced growth path $\dot{k}/k = g$. So in steady state

$$g = s \frac{y}{k} - (\delta + n). \quad (11)$$

Plug in for y to get

$$g = s \frac{(Ah)^{1-\alpha}}{k^{1-\alpha}} - (\delta + n). \quad (12)$$

Solve for

$$\frac{k}{Ah} = \left(\frac{s}{\delta + n + g} \right)^{1/(1-\alpha)}. \quad (13)$$

Output per Worker Level

Given

$$\frac{k}{Ah} = \left(\frac{s}{\delta + n + g} \right)^{1/(1-\alpha)} . \quad (14)$$

we know that

$$y = Ah \left(\frac{k}{Ah} \right)^\alpha \quad (15)$$

$$y = Ah \left(\frac{s}{\delta + n + g} \right)^{\alpha/(1-\alpha)} \quad (16)$$

$$y(t) = A(t)e^{\psi u} \left(\frac{s}{\delta + n + g} \right)^{\alpha/(1-\alpha)} . \quad (17)$$

We see here that human capital, as determined by u , influences the level of output per worker, even though it does not change the growth rate of output per worker.

Relative Output per Worker

Consider the model in relative terms. Relative to a rich-country standard like the U.S.

$$\hat{y}_i = \frac{y_i}{y_{US}} \quad (18)$$

so that \hat{y}_i is the output per worker of country i relative to that in the U.S.

If output per worker is described as in our modified model, then

$$\hat{y}_i = \frac{A_i e^{\psi u_i} \left(\frac{s_i}{\delta + n_i + g} \right)^{\alpha/(1-\alpha)}}{A_{US} e^{\psi u_{US}} \left(\frac{s_{US}}{\delta + n_{US} + g} \right)^{\alpha/(1-\alpha)}}. \quad (19)$$

which can reduce to

$$\hat{y}_i = \frac{A_i}{A_{US}} e^{\psi(u_i - u_{US})} \left(\frac{s_i}{s_{US}} \right)^{\alpha/(1-\alpha)} \left(\frac{\delta + n_{US} + g}{\delta + n_i + g} \right)^{\alpha/(1-\alpha)} \quad (20)$$

Note, we've made the assumption that g is identical for all countries.

Explaining Cross-Country Variation

Solow originally assumed that A was identical across countries, as they could share technology. How good does of a job does the model do if $A_i = A_{US}$?

$$\hat{y}_i = e^{\psi(u_i - u_{US})} \left(\frac{s_i}{s_{US}} \right)^{\alpha/(1-\alpha)} \left(\frac{\delta + n_{US} + g}{\delta + n_i + g} \right)^{\alpha/(1-\alpha)} \quad (21)$$

Are the differences in u , s , and n sufficient to explain cross-country output per worker differences?

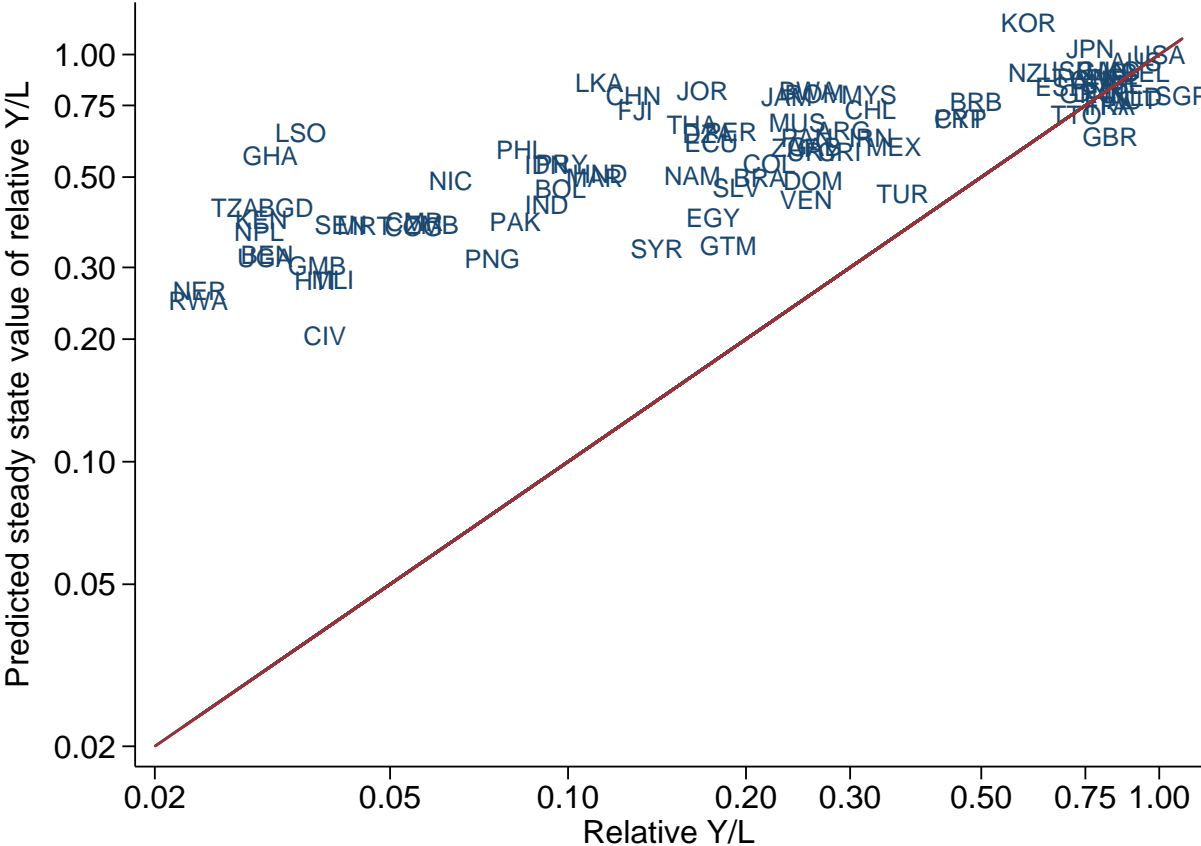
Plug in values of $\alpha = 1/3$, $\psi = 0.10$, $\delta + g = 0.075$. Use years of education as u_i . Use average savings rate as s_i . Use average population growth as n_i .

Example: $s_{India} = 0.241$, $u_{India} = 4.23$, $n_{India} = 0.017$. $s_{US} = 0.202$, $u_{US} = 13.24$, $n_{US} = 0.011$. So

$$\hat{y}_{India} = e^{0.10(4.23 - 13.24)} (0.241/0.202)^{1/2} (0.086/0.092)^{1/2} = 0.429 \quad (22)$$

Based on education, savings, and population growth, India should be 43% as rich as U.S. India is actually about 9% as rich as U.S.

All Countries



The Solow Residual

Savings, education, and population growth do not explain all of the variation in output per worker. Those three factors suggest most poor countries should be much better off than they actually are. So what's missing?

Technology/productivity differences. $A_i \neq A_{US}$ for countries. We cannot measure A_i directly, but we can infer it from data. Take

$$y_i = k_i^\alpha (A_i h_i)^{1-\alpha} \quad (23)$$

and re-arrange to

$$A_i = \left(\frac{y_i}{k_i^\alpha h_i^{1-\alpha}} \right)^{1/(1-\alpha)}. \quad (24)$$

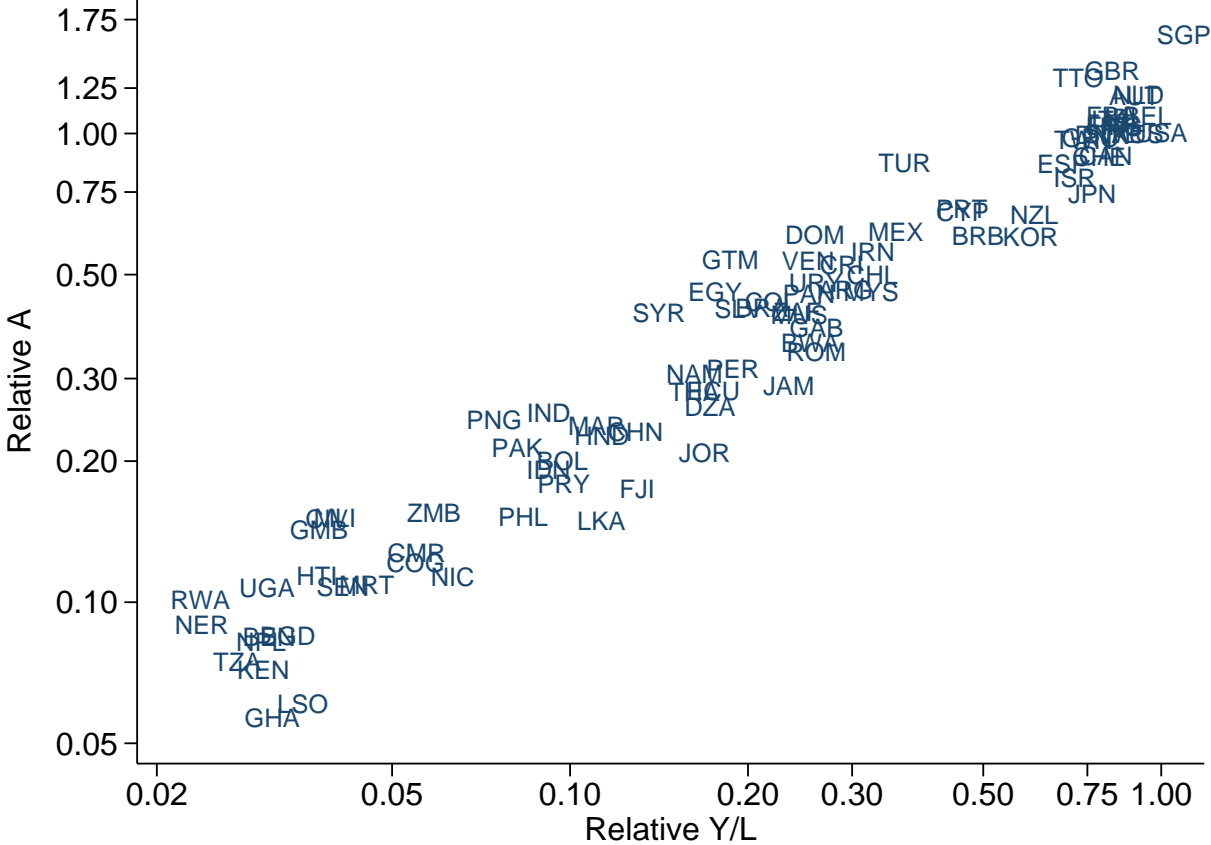
Given data on y_i , k_i , and h_i we can back out the actual value of A_i .

The value of A_i from this is sometimes called “The Solow Residual”. It measures everything that matters besides k_i , h_i for output per worker.

Last, for comparison, calculate

$$\hat{A}_i = \frac{A_i}{A_{US}} \quad (25)$$

Values of \hat{A}_i across countries



Technology Drives Differences

The values of \hat{A}_i , by themselves, do a good job of describing differences in output per worker.

Differences in A_i explain about 1/2 to 2/3 of the differences in output per worker across countries.

Why we will focus on explanations of the growth in A_i and/or the level of A_i rather than explanations of why s_i or n_i varies.

Growth Rate Differences

Some countries grow more quickly than others. Why?

One explanation: *convergence*. Poor countries grow more quickly than rich countries.

Look at equation for growth rate of \dot{k}/k .

$$\frac{\dot{k}}{k} = s \frac{y}{k} - (\delta + n) \quad (26)$$

$$= s \left(\frac{Ah}{k} \right)^{1-\alpha} - (\delta + n) \quad (27)$$

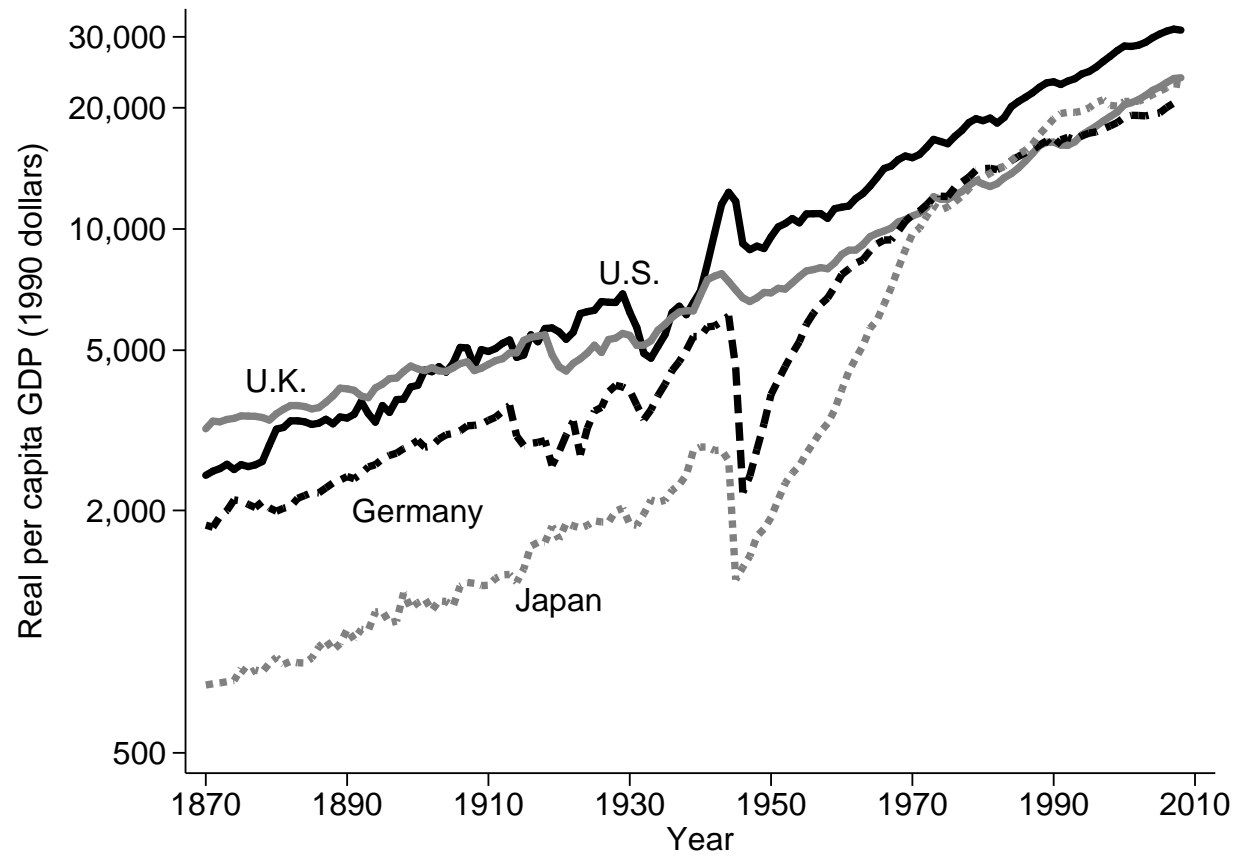
if

$$\frac{k}{Ah} < \left(\frac{s}{\delta + n + g} \right)^{1/(1-\alpha)} \quad (28)$$

then $\dot{k}/k > g$. So if k/Ah is low relative to steady state, country grows quickly.

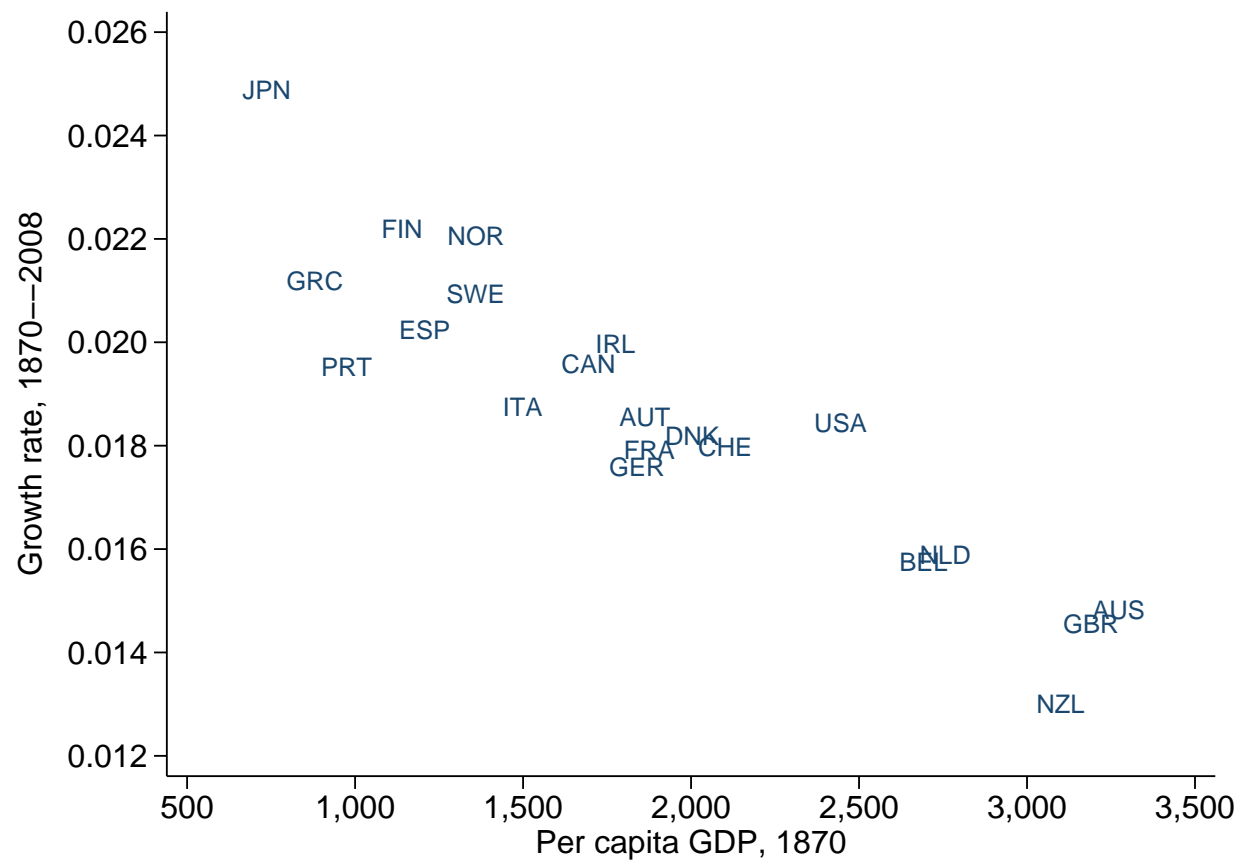
Long-run Convergence

Countries with similar A and h :



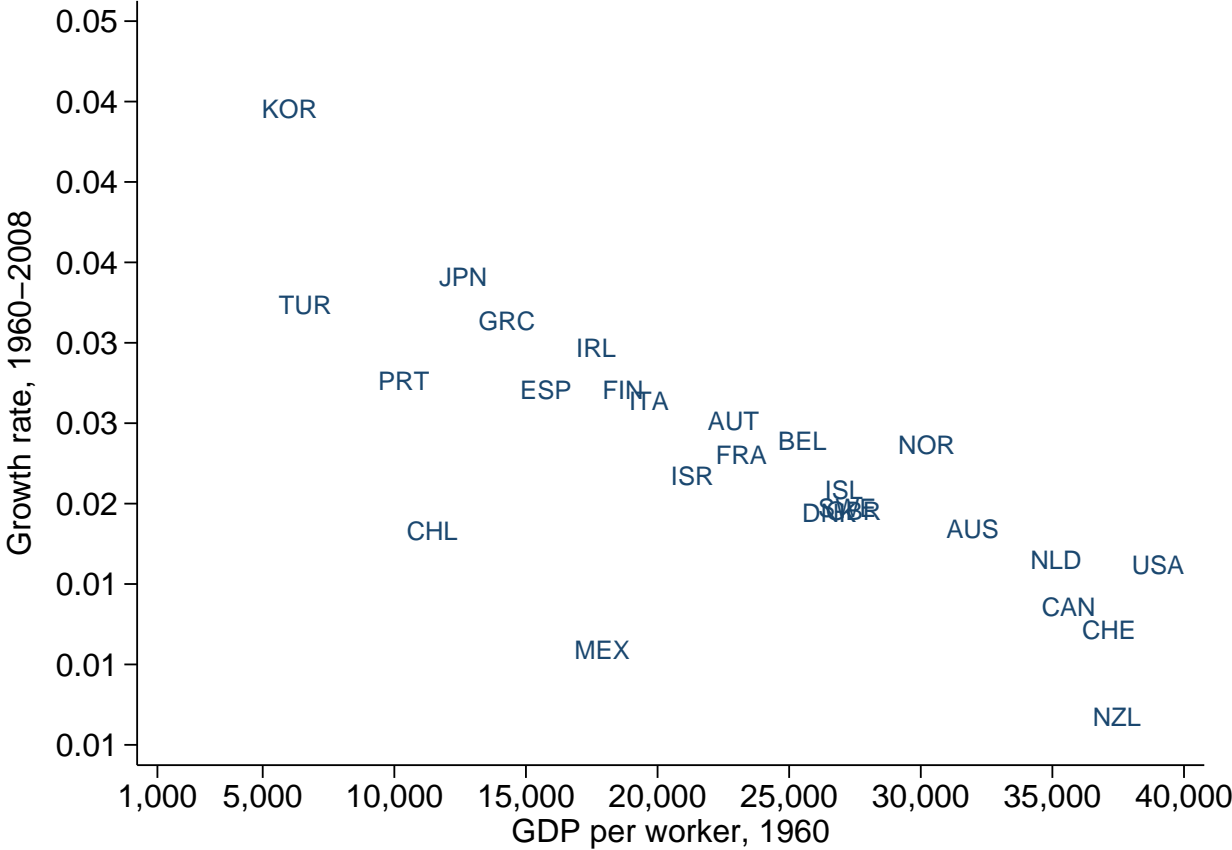
Long-run Convergence

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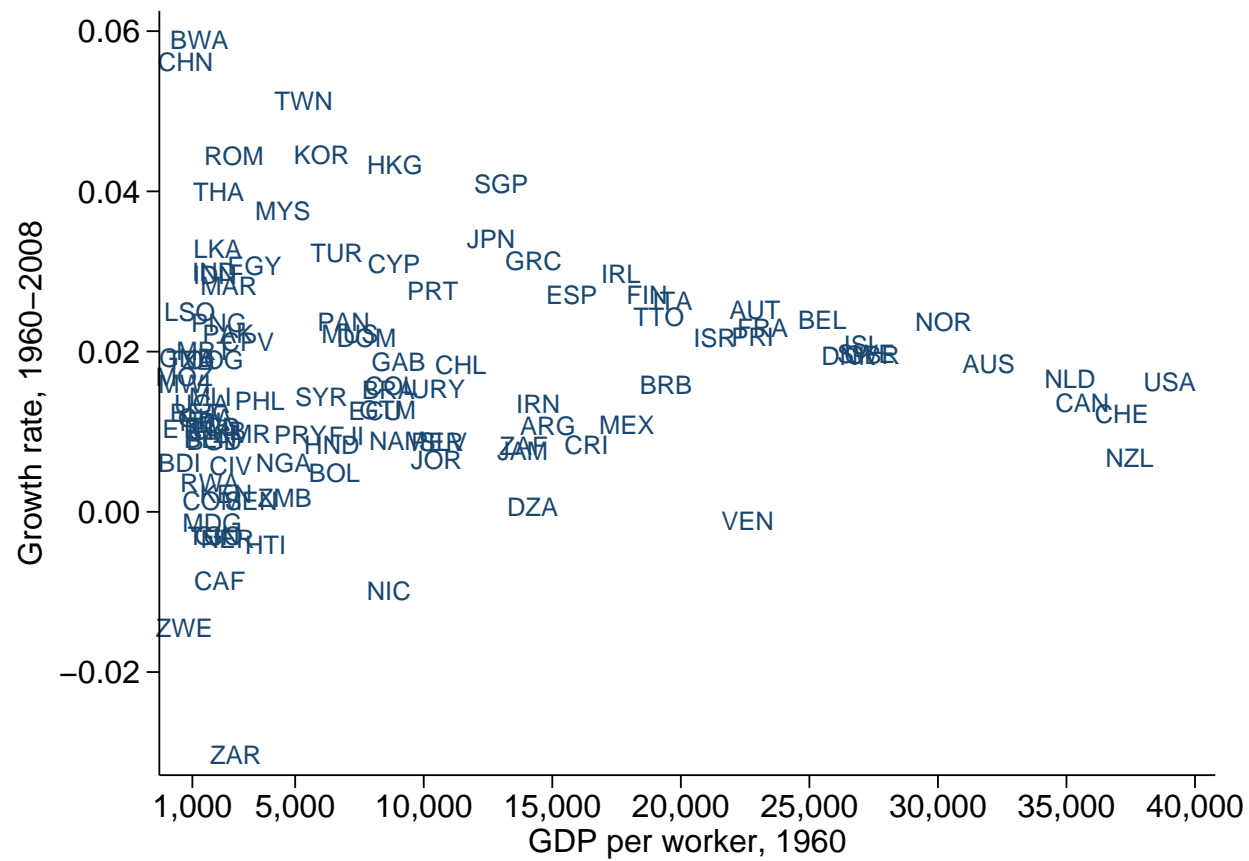
Convergence in the OECD

Countries with similar A and h :



Lack of Convergence

Countries with dis-similar A and h :



Conditional Convergence

Not all countries seem to fit. They grow very slowly even though they are poor.

Conditional convergence: countries grow faster, the farther they are *from their own steady state*. Poor countries have low steady states, so they are already close to their steady state, and grow slowly.

Can we see this in the data? Compute the steady state for each country as

$$y_i^* = A_i h_i \left(\frac{s_i}{\delta + g + n_i} \right)^{\alpha/(1-\alpha)} \quad (29)$$

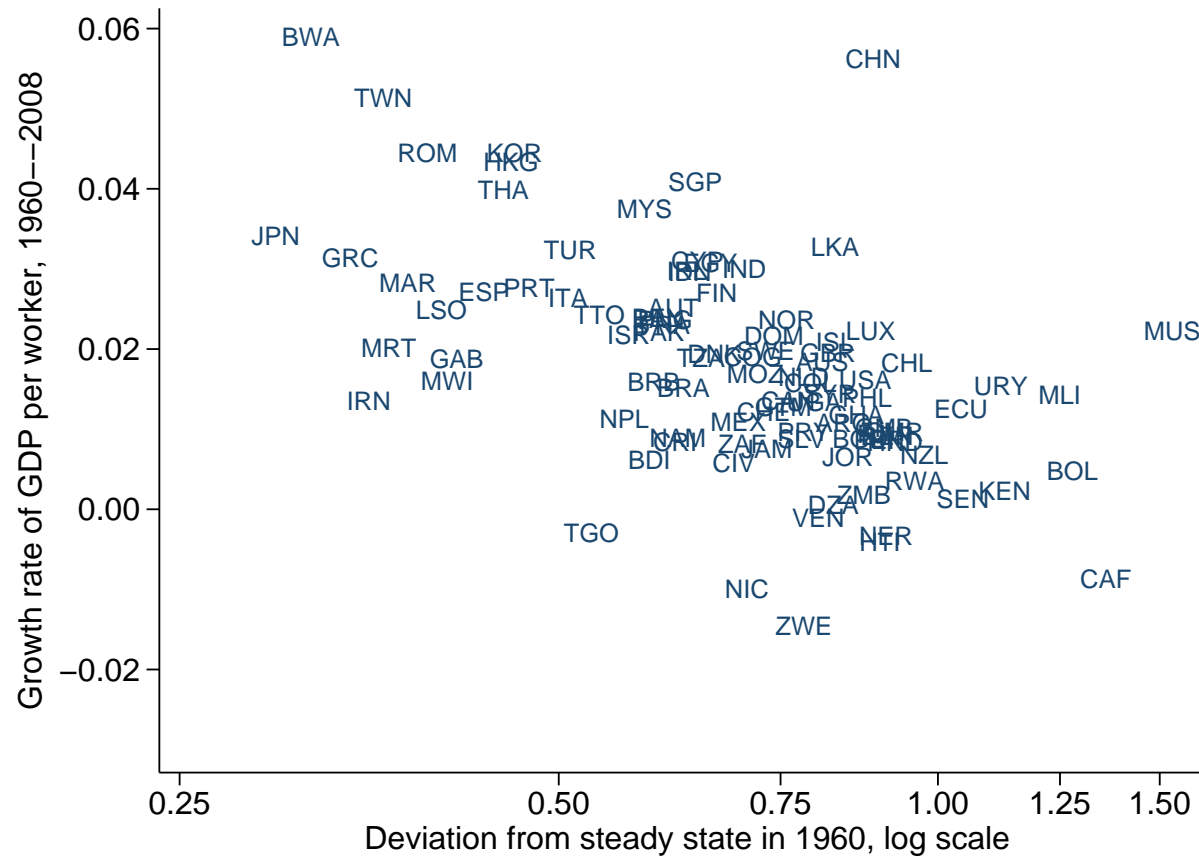
using data on h_i , s_i , and n_i like before. Use the value of A_i from 1970.

Then compute how far each country is from steady state

$$\frac{y_i}{y_i^*} \quad (30)$$

and graph growth from 1960–2008 versus this relative value.

Lack of Convergence

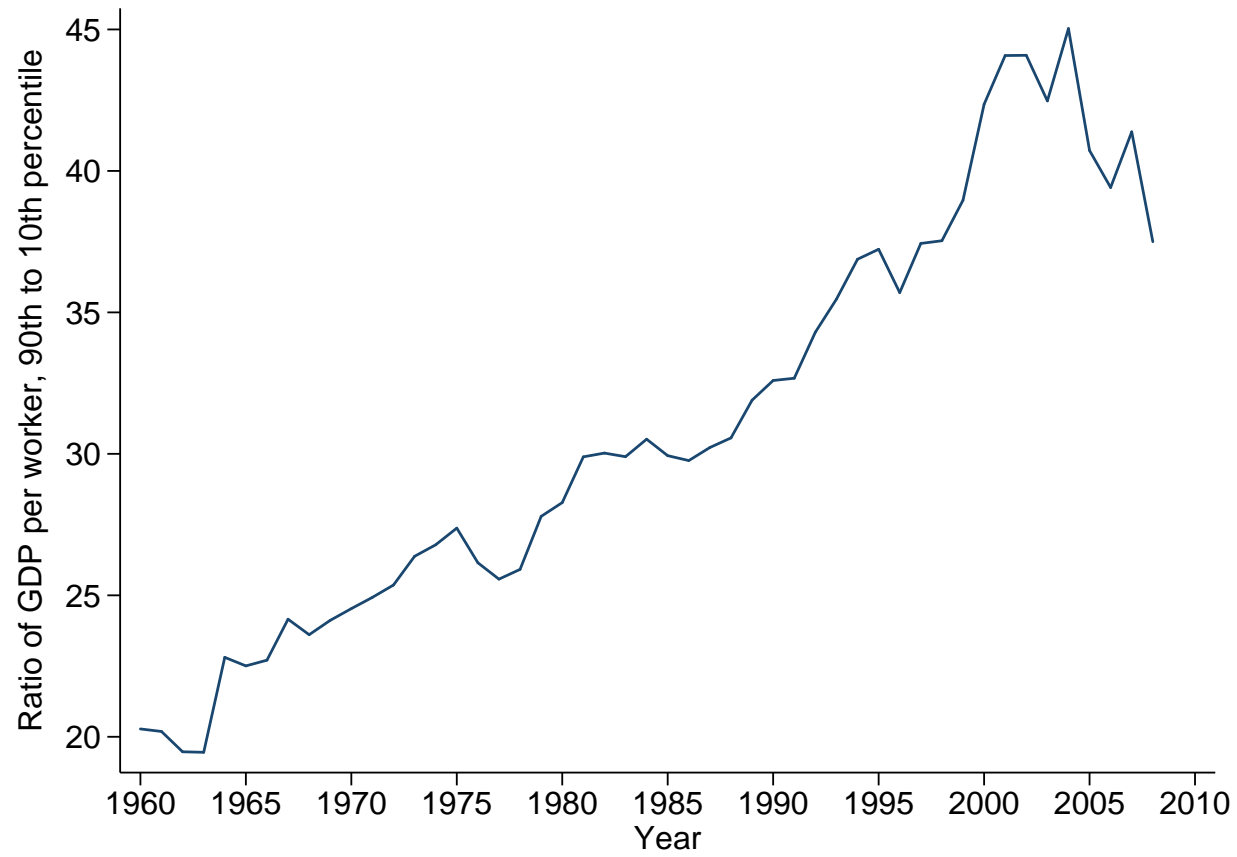


Equality?

If there were absolute convergence (like in the OECD), then countries are getting more equal. If there is conditional convergence (like in the whole sample), then countries may be getting more unequal.

In absolute numbers of people, world looks like it is getting more equal. Sala-i-Martin (2006): In 1970, 534 million people (15% of world population) lived on less than \$1 per day. In 2000 only 321 million people (6% of world population) lived on less than \$1.

Lack of Convergence



Lack of Convergence

